Homework 2 Solutions Chapter 2 # 2, 3, 5, 6, 8, 9, 12, 17, 18, 20

#2 Let $X = \{1,2,3,4,5\}$. The sample space S consists of all finite sequences $(x_1, x_2, \ldots, x_n)$ where $x_1, x_2, \ldots, x_{n-1} \in X$ and $x_n = 6$, as well as the infinite set of sequences with values in X.

The event E_n is the set of all sequences of length n in S.

$\left(\bigcup_1^{\infty} E_n\right)^c$ is the set of all infinite sequences $(x_1, x_2, \ldots, x_n, \ldots)$ with

$x_i \in X = \{1,2,\ldots,5\}$ for $i = 1,2,\ldots,n,\ldots$

#3 $E = \{(1,2),(1,4),(1,4),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),(4,1),$
$(4,3),(4,5),(5,2),(5,4),(5,6),(6,1),(6,3),(6,5)\}$

$F = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$

$G = \{(1,4),(2,3),(3,2),(4,1)\}$

$E \cup F$: Sum is odd or at least one of the dice lands on 1

$E \cap F$: Sum is odd and at least one of the dice lands on 1

FG: At least one of the dice lands on 1 and the sum equals 5.

Thus $FG = \{(1,4),(4,1)\}$

EF^c: The sum is odd and none of the dice lands on 1.

EFG: The sum is odd and one of the dice lands on 1 and the sum is five. Notice that $EFG = FG$

#5 a) 2^5 outcomes by the counting principle

b) $W = \{11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111,$

$00110, 00111, 01110, 01111, 10110, 10111, 10101\}$

c) Fix components 4 and 5. There are $2 \cdot 2 \cdot 2$ ways to fill in the other three spots.

d) $A = \{10000, 11000, 11100, 10100, 00000, 01000, 00100, 00100\}$

Thus $A \cap W = \{11100, 11000\}$

#6 a) $S = \{(0,g), (0,f), (0,s), (1,g), (1,f), (1,s)\}$

b) $A = \{(0,s), (1,s)\}$

c) $B = \{(0,g), (0,f), (0,s)\}$

d) $B^c \cup A = \{(1,g), (1,f), (1,s), (0,s)\}$

#8 a) $P(A \cup B) = P(A) + P(B)$ since A, B are mutually exclusive.

Thus $P(A \cup B) = 0.8$

b) Since $A \cap B = \emptyset$ we have that $A \cap B^c = A$.

Thus $P(A \cap B^c) = P(A) = 0.3$

c) $P(A \cap B) = P(\emptyset) = 0$

Recall the proof that $P(\emptyset) = 0$. We have $\emptyset \cup \emptyset = \emptyset$ and

thus $P(\emptyset \cup \emptyset) = P(\emptyset) \Rightarrow P(\emptyset) + P(\emptyset) = P(\emptyset) \Rightarrow P(\emptyset) = 0$.

#9 Let A is the event that a randomly chosen customer has Amex

and B -‖- has Visa

We want $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.24 + 0.61 - 0.11 = 0.74$

Thus 74% carry a credit card that the establishment will accept.

#12 a) S is the event that a randomly chosen student is taking Spanish

F -‖- French

G -‖- German

$P(S) = \frac{28}{100}$, $P(F) = \frac{26}{100}$, $P(G) = \frac{16}{100}$

$P(SF) = \frac{12}{100}$, $P(SG) = \frac{4}{100}$, $P(FG) = \frac{6}{100}$, $P(SFG) = \frac{2}{100}$

We want $1 - P(S \cup F \cup G) = 1 - P(S) - P(F) - P(G) - P(SFG)$
$+ P(SF) + P(SG) + P(FG) = 0.5$

b) We want $P(\text{exactly in one language class})$

$= P(\text{only in } S) + P(\text{only in } F \text{ class}) + P(\text{only in } G \text{ class})$

But $P(\text{only in } S) = P(S) - P(SF) - P(SG) + P(SFG)$

$$= \frac{28}{100} - \frac{12}{100} - \frac{4}{100} + \frac{2}{100} = \frac{14}{100} = 0.14$$

Similarly $P(\text{only in } F) = 0.1$

$P(\text{only in } G) = 0.08$ and thus

$P(\text{exactly one language}) = 0.14 + 0.1 + 0.08 = 0.32$

c) The total number of students that don't take a language class is 50. From part a) we know that $1 - P(S \cup F \cup G) = 0.5$ and thus

students that are not in any language class $= 100 \times 0.5 = 50$

Thus $\dfrac{\binom{50}{2}}{\binom{100}{2}} = \dfrac{49}{198}$ is the probability that neither of the selected students are in a language class and thus

$1 - \dfrac{49}{198} = \dfrac{149}{198}$ is the probability that at least 1 is taking a class.

#17 The chess board has 64 squares. Thus there

$64 \cdot 63 \cdot 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57$ ways of arranging 8 castles on a chess board.

Now there are 64 ways to put the first castle. Now excluding the row and column that this particular castle is, there are $7^2 = 49$ ways to put the second. Excluding his row and column, there are $6^2 = 36$ ways to put the third, etc.

Thus there are $8^2 7^2 6^2 5^2 4^2 3^2 2^2 1^2 = \prod_{i=1}^{8} i^2$ ways in which none of the castles can capture any of the other. Thus the probability is

$$P = \frac{\prod_{i=1}^{8} i^2}{64 \cdot 63 \cdot 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57}$$

#18 The number of total outcomes is $52 \cdot 51$.

If I first pick the ace, there are 4 (four aces) × 16 (four tens, four jacks, four queens and four kings) ways to form a blackjack. But there are also

16×4 ways to pick any of the 4 tens, 4 jacks, 4 queens and 4 kings following by an ace.

Thus $$P = \frac{4 \cdot 16 + 16 \cdot 4}{52 \cdot 51} \approx 0.048$$

(#20) From #18 we know that if A is the event that you are dealt a blackjack, and B the event that the dealer is dealt a blackjack we have

$$P(A) = \frac{4\cdot 16 + 16\cdot 4}{52\cdot 51} \qquad P(B) = \frac{4\cdot 16 + 16\cdot 4}{52\cdot 51}$$

and similar logic shows that $P(A \cap B) = \dfrac{(4\cdot 16 + 16\cdot 4)(3\cdot 15 + 15\cdot 3)}{52\,51\,50\,49}$

$$= \frac{4\cdot 4\cdot 16\cdot 3\cdot 15}{52515049}$$

The probability that neither you nor the dealer is dealt a blackjack is

$$1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 0.9052$$