Chapter 2

HW 3 Solutions Pg 53-54 # 32, 37, 41, 43, 46, 49, 53

Chapter 3 Pg 102 # 1, 5, 6

#32 Number in the sample space is $(b+g)!$
Put a girl in the ith position. There are $(b+g-1)!$ possibilities for the rest. Since there are g girls we have

$$P = \frac{g(b+g-1)!}{(b+g)!} = \frac{g}{b+g}$$

#37 a) There are $\binom{10}{5}$ selections for the final exam

The number of selections that allow the student to solve all problems is $\binom{7}{5}$ and thus $P = \frac{\binom{7}{5}}{\binom{10}{5}} = 0.0833$

b) There are $\binom{7}{4}\binom{3}{1}$ selections that allow the student to solve exactly four problems. Thus the probability of solving at least 4 is $P = \frac{\binom{7}{5}+\binom{7}{4}\binom{3}{1}}{\binom{10}{5}} = \frac{1}{2}$

#41 Let P the probability that 6 doesn't show up at all

Then $P = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6} = \left(\frac{5}{6}\right)^4$

Thus the probability that 6 comes at least one is

$$1-\left(\frac{5}{6}\right)^4 = 0.5177$$

Second way: Let E_i, $i=1,2,3,4$ the event that 6 comes in the ith role.

Then we use the inclusion/exclusion principle and calculate

$$P(E_1 \cup E_2 \cup E_3 \cup E_4) = \sum_{i=1}^{4} P(E_i) - \sum_{l_1<l_2} P(E_{l_1} E_{l_2}) + \sum_{l_1<l_2<l_3} P(E_{l_1} E_{l_2} E_{l_3}) - \sum_{l_1<l_2<l_3<l_4} P(E_{l_1} \dots E_{l_4})$$

Since $P(E_i) = \frac{1}{6}$, $P(E_{l_1} E_{l_2}) = \frac{1}{6^2}$, ... $P(E_1 E_2 E_3 E_4) = \frac{1}{6^4}$

Then $$P\left(\bigcup_{i=1}^{4} E_i\right) = \frac{4}{6} - \binom{4}{2}\frac{1}{6^2} + \binom{4}{3}\frac{1}{6^3} - \binom{4}{4}\frac{1}{6^4}$$

$$= \frac{4}{6} - \frac{6}{36} + \frac{4}{36\times 6} - \frac{4}{6^4} = 0.5177$$

#43 a) There are $N!$ ways to arrange N people in a line. There are $2(N-1)!$ ways to arrange them such that A and B sit to each other.

Thus $$P = \frac{2(N-1)!}{N!} = \frac{2}{N}$$

b) If A peaks a seat there are $N-1$ seats left, two of which are next to A. Thus $P = \frac{2}{N-1}$.

Another way to argue is that for every position of A (N positions on the circle) there are two ways of assigning B next to A and then permute the rest $(N-2)!$

Thus in total $2N(N-2)!$ arrangements and the probability

is $P = \dfrac{2N(N-2)!}{N!} = \dfrac{2}{N-1}$.

\# 46 First evaluate the probability that no two of them celebrate their

birthday on the same month. There are 12^n total outcomes, $n \leq 12$

Then the probability is $\dfrac{12 \cdot 11 \cdot 10 \cdots (12-n+1)}{12^n}$

Thus $P(n) = 1 - \dfrac{12 \cdot 11 \cdot 10 \cdots (12-n+1)}{12^n}$ is the probability that at least

two celebrate their birthday in the same month.

We want to find n such that $P(n) \geq \frac{1}{2}$ or

$$\frac{1}{2} \geq \frac{12 \cdot 11 \cdots (12-n+1)}{12^n}$$

If $n=1$ $\quad \frac{1}{2} \not\geq 1$, $n=2$ $\quad \frac{1}{2} \not\geq \frac{12 \cdot 11}{12^2} = \frac{11}{12}$

$n=3$ $\quad \frac{1}{2} \not\geq \dfrac{12 \cdot 11 \cdot 10}{12^3} = \dfrac{11 \cdot 10}{12 \cdot 12}$

$n=4$ $\quad \frac{1}{2} \not\geq \dfrac{11 \cdot 10 \cdot 9}{12 \cdot 12 \cdot 12} = \dfrac{11 \cdot 5 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot 6 \cdot \cancel{3} \cdot 4 \cdot \cancel{3} \cdot 4} = \dfrac{55}{96}$

But for $n=5$, $\dfrac{11 \cdot 10 \cdot 9 \cdot 8}{12 \cdot 12 \cdot 12 \cdot 12} = \dfrac{11 \cdot \cancel{2} \cdot 5 \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{4}}{\cancel{2} \cdot 6 \cdot \cancel{3} \cdot 4 \cdot \cancel{3} \cdot \cancel{4} \cdot 3 \cdot \cancel{4}_{\,2}} = \dfrac{55}{144}$

Thus for $n=5$, $P(5) = 1 - \frac{55}{144} \geq \frac{1}{2}$

#49 This is easy.

$$P = \frac{\binom{6}{3}\binom{6}{3}}{\binom{12}{6}} \cong 0.433$$

#53 This is similar with example 5n on page 42 in the book the only difference being that the couples are arranged in a row instead of a circle.

Thus if E_i is the event that the i-th couple sit together for $i=1,2,3,4$

We want $1 - P(E_1 \cup E_2 \cup E_3 \cup E_4)$.

But $P(E_1 \cup E_2 \cup E_3 \cup E_4) = \sum_{i=1}^{4} P(E_i) - \sum_{i<j} P(E_i E_j) + \sum_{i<j<k} P(E_i E_j E_k) - P(E_1 E_2 E_3 E_4)$

But as in the example in the book $P(E_{i_1}, E_{i_2} \ldots E_{i_n}) = \frac{2^n (8-n)!}{8!}$

Thus $P(E_i) = \frac{2}{8}$, $P(E_i E_j) = \frac{2^2\, 6!}{8!}$, $P(E_i E_j E_k) = \frac{2^3\, 5!}{8!}$ and

$P(E_1 E_2 E_3 E_4) = \frac{2^4\, 4!}{8!}$

Thus $P(E_1 \cup E_2 \cup E_3 \cup E_4) = 4 \cdot \frac{2}{8} - \binom{4}{2} \frac{2^2\, 6!}{8!} + \binom{4}{3} \frac{2^3\, 5!}{8!} - \frac{2^4\, 4!}{8!}$

$= 1 - \frac{3}{7} + \frac{2}{21} - \frac{1}{105} = \frac{23}{35}$

and thus $P = 1 - \frac{23}{35} = \frac{12}{35} \cong 0.3428$

#1 Let E be the event that at least one lands on 6
Let F be the event that the dice land on different numbers.

Thus $P(E|F) = \frac{P(EF)}{P(F)}$

But $P(F) = \frac{30}{36}$ (since there are 6 outcomes that the dice land on the same numbers)

To find $P(EF)$ note that $\{(1,6),(2,6),(3,6),(4,6),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5)\}$ is the event EF.

Thus $P(EF) = \frac{10}{36}$ and

$$P(E|F) = \frac{10/36}{30/36} = \frac{1}{3}$$

#5 $$P = \frac{6 \cdot 5 \cdot 9 \cdot 8}{15 \cdot 14 \cdot 13 \cdot 12}$$

#6 a) With replacement the possible outcomes are

(WWWR, WWRW, WRWW, RWWW) thus we pick 4 of which 3 are white.

The total number of outcomes in the sample space is

$$\frac{8}{12}\frac{8}{12}\frac{8}{12}\frac{4}{12}+\frac{8}{12}\frac{8}{12}\frac{4}{12}\frac{8}{12}+\frac{8}{12}\frac{4}{12}\left(\frac{8}{12}\right)^2+\frac{4}{12}\left(\frac{8}{12}\right)^3=4\cdot\frac{4}{12}\left(\frac{8}{12}\right)^3$$

If the first and third are white we have the events

WWWR and WRWW. Thus

$$P=\frac{2\cdot\frac{4}{12}\left(\frac{8}{12}\right)^3}{4\,\frac{4}{12}\left(\frac{8}{12}\right)^3}=\frac{1}{2}$$

b) Without replacement let A the event that the sample contains exactly three white balls. Since WWRW, WRWW, WWWR, RWWW we have that $P(A)=4\,\frac{8\cdot7\cdot6\cdot4}{12\cdot11\cdot10\cdot9}$

Let B the event that the first and third are white. Again WRWW, WWWR

and $P(AB)=2\,\frac{8\cdot7\cdot6\cdot4}{12\cdot11\cdot10\cdot9}$

and $P(B|A)=\frac{P(AB)}{P(A)}=\frac{1}{2}$.