

Solutions

(1)

HW 4 Chapter 3 # 9, 10, 20, 23, 30, 42, 43, 51, 5F, 64

#9 Let K be the event that the ball chosen from urn A is white and L the event that exactly 2 white balls are chosen.

We look for $P(K|L) = \frac{P(KL)}{P(L)}$

KL the event that the ball from A ^(is white) and that exactly 2 white balls are chosen

$$P(KL) = \frac{2}{6} \frac{4}{12} \frac{1}{4} + \frac{2}{6} \frac{8}{12} \frac{3}{4} = \frac{1}{36} + \frac{1}{6} = \frac{7}{36}$$

$$P(L) = \frac{2}{6} \frac{4}{12} \frac{1}{4} + \frac{2}{6} \frac{8}{12} \frac{3}{4} + \frac{4}{6} \frac{8}{12} \frac{1}{4} = \frac{7}{36} + \frac{1}{9} = \frac{11}{36}$$

$$P(K|L) = \frac{\frac{7}{36}}{\frac{11}{36}} = \frac{7}{11}$$

#10 Let R_i the event that the i th card is a spade, $i=1,2,3$

We want $P(R_1 | R_2 R_3) = \frac{P(R_1 R_2 R_3)}{P(R_2 R_3)}$

$$P(R_1 R_2 R_3) = \frac{13}{52} \frac{12}{51} \frac{11}{50}$$

Now $R_2 R_3 = R_2 R_3 R_1 \cup R_2 R_3 R_1^c \Rightarrow P(R_2 R_3) = P(R_2 R_3 R_1) + P(R_2 R_3 R_1^c)$

$$= \frac{13}{52} \frac{12}{51} \frac{11}{50} + \frac{52-13}{52} \cdot \frac{13}{52} \cdot \frac{12}{51}$$

$$\text{and } P(R_1, R_2, R_3) = \frac{13 \cdot 12 \cdot 11}{52 \cdot 51 \cdot 50} = \frac{13 \cdot 12 \cdot 11}{13 \cdot 12 \cdot 11 + 39 \cdot 13 \cdot 12} = \frac{11}{50} = 0.22$$

#20 let A be the event that a student is female. we know $P(A) = 0.52$
 let B be the event that a student is majoring in CS. Then $P(B) = 0.05$
 We also know that $P(AB) = 0.02$

a) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{0.02}{0.05} = \frac{2}{5}$

b) $P(B|A) = \frac{P(AB)}{P(A)} = \frac{0.02}{0.52} = \frac{1}{26}$

#23 a) let E be the event that the ball from urn II is white.
 F - II - I is white

$$E = EF \cup EF^c \Rightarrow P(E) = P(EF) + P(EF^c) = P(E|F)P(F) + P(E|F^c)P(F^c)$$

But $P(\text{ball from II white given ball from I white}) = \frac{2}{3} = P(E|F)$

Similarly $P(E|F^c) = \frac{1}{3}$. $P(F) = \frac{1}{3}$, $P(F^c) = \frac{2}{3}$

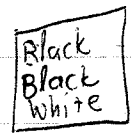
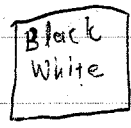
Thus $P(E) = \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} = \frac{4}{9}$

b) We want $P(F|E) = \frac{P(EF)}{P(E)} = \frac{P(E|F)P(F)}{P(E)} = \frac{\frac{2}{3} \cdot \frac{1}{3}}{\frac{4}{9}} = \frac{1}{2}$

#30 Let A the event that the selected marble is black. We want

$P(A)$. Let F the event that the selected box is the one with 1 black, 1 white and the F^c - (1) - (1) - 2 black, 1 white

$$P(A) = P(A|F)P(F) + P(A|F^c)P(F^c)$$



$$P(A|F) = \frac{1}{2}, \quad P(A|F^c) = \frac{2}{3}$$

$$P(F) = P(F^c) = \frac{1}{2}$$

$$\text{Thus } P(A) = \frac{1}{2} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

$$\text{We want } P(F|A^c) = \frac{P(F|A^c)}{P(A^c)} = \frac{P(A^c|F)P(F)}{P(A^c)}$$

$$P(A^c|F) = \frac{1}{2} \quad \text{thus } P(F|A^c) = \frac{\frac{1}{2} \cdot \frac{1}{2}}{1 - \frac{7}{12}} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5}$$

#42 F_1 the event that the cake was baked by A $P(F_1) = 0.5$
 F_2 - (1) - by B $P(F_2) = 0.3$
 F_3 - (1) - by C $P(F_3) = 0.2$

Let E be the event that the cake failed to rise.

Then $P(F_1|E) = P(\text{given that the cake failed to rise, the event that it was baked by A}) = \text{proportion of failure caused by A}$

$$\text{But } P(E) = P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + P(E|F_3)P(F_3)$$

$$= 0.02 \times 0.5 + 0.03 \cdot 0.3 + 0.05 \times 0.2 = 0.029$$

Thus $P(F_1|E) = \frac{P(E|F_1)P(F_1)}{P(E)} = \frac{0.02 \cdot 0.5}{0.029} = \frac{10}{29}$

- #43 F_1 the event that the 2 headed coin selected
- F_2 the event that the fair coin selected
- F_3 -11- that the biased coin selected

First calculate the probability that the coin shows head.

$$P(E) = P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + P(E|F_3)P(F_3)$$

$$= 1 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{4} = \frac{3}{4}$$

We want $P(F_1|E) = \frac{P(E|F_1)P(F_1)}{P(E)} = \frac{\frac{1}{3}}{\frac{3}{4}} = \frac{4}{9}$

#51 We have $P(\text{got job} | \text{Strong Recommendation}) = 0.8$

$$P(\text{got the job} | \text{Moderate Recommendation}) = 0.4$$

$$P(\text{got the job} | \text{Weak Recommendation}) = 0.1$$

$$P(\text{Strong Rec.}) = 0.7, \quad P(\text{Moderate Rec.}) = 0.2, \quad P(\text{Weak Rec.}) = 0.1$$

a) Thus $P(\text{got the job}) = 0.8 \times 0.7 + 0.4 \times 0.2 + 0.1 \times 0.1 = 0.65$

b) $P(\text{Strong} | \text{got the job}) = \frac{P(\text{got the job} | \text{Strong}) P(\text{Strong})}{P(\text{got the job})}$

Thus $P(\text{Strong} | \text{Got the job}) = \frac{0.8 \times 0.7}{0.65} = \frac{56}{65}$

c) $P(\text{Strong} | (\text{got the job})^c) = \frac{P((\text{got the job})^c | \text{Strong}) P(\text{Strong})}{P((\text{got the job})^c)}$
 $= \frac{0.2 \times 0.7}{1 - 0.65} = \frac{14}{35}$

Similarly $P(\text{Moderate} | J^c) = \frac{0.6 \times 0.2}{1 - 0.65} = \frac{12}{35}$

$P(\text{Weak} | J^c) = \frac{9}{35}$

#57 The daily movements are independent events. After 2 days has the same value if the first day went up and then down or vice versa.

a) Thus $P(\{U_p \text{Down}, \text{Down} U_p\}) = P(U_p \text{Down}) + P(\text{Down} U_p)$
 $= P(U_p) P(\text{Down}) + P(\text{Down}) P(U_p) = 2 P(U_p) P(\text{Down}) = 2P(1-P)$

b) $P(\{U_p \text{Down} U_p, U_p U_p \text{Down}, \text{Down} U_p U_p\}) = 3 P(U_p) P(\text{Down})$
 $= 3P^2(1-P)$

c) $P(\underset{A}{U_p \text{ on first day}} | \text{Given that in } \underset{B}{3 \text{ days increased by 1 unit}})$

$P(A|B) = \frac{P(B|A) P(A)}{P(B)} = \frac{(P(1-P) + (1-P)P) \times P}{3 P^2(1-P)} = \frac{2}{3}$

since $P(B|A) = 2P(1-P)$

(#64) a) let H the event that the husband is chosen and let E the event that they get the question right.

$$\text{Then } P(E) = P(E|H)P(H) + P(E|H^c)P(H^c) = P \times \frac{1}{2} + P \times \frac{1}{2} = P$$

b) let E the event that they get the question right, let H_1 the event that they agree on the correct answer, H_2 the event that they agree on the incorrect answer and H_3 the event that they disagree.

$$\begin{aligned} \text{Then } P(E) &= P(E|H_1)P(H_1) + P(E|H_2)P(H_2) + P(E|H_3)P(H_3) = \\ &= 1 \times P^2 + 0 \times (1-P)^2 + \frac{1}{2} \times 2P(1-P) \\ &= P^2 + P(1-P) = P. \end{aligned}$$

Thus it doesn't matter which strategy to choose.