Solutions

HW 4 Chapter 3 # 9, 10, 20, 23, 30, 42, 43, 51, 57, 64

#9 Let K be the event that the ball chosen from urn A is white and L the event that exactly 2 white balls are chose.

We look for $P(K|L) = \dfrac{P(KL)}{P(L)}$

KL the event that the ball from A is white and that exactly 2 white balls are chosen

$$P(KL) = \frac{2}{6}\,\frac{4}{12}\,\frac{1}{4} + \frac{2}{6}\,\frac{8}{12}\,\frac{3}{4} = \frac{1}{36} + \frac{1}{6} = \frac{7}{36}$$

$$P(L) = \frac{2}{6}\,\frac{4}{12}\,\frac{1}{4} + \frac{2}{6}\,\frac{8}{12}\,\frac{3}{4} + \frac{4}{6}\,\frac{8}{12}\,\frac{1}{4} = \frac{7}{36} + \frac{1}{9} = \frac{11}{36}$$

$$P(K|L) = \frac{7/36}{11/36} = \frac{7}{11}$$

#10 Let R_i the event that the ith card is a spade, $i=1,2,3$

We want $P(R_1|R_2R_3) = \dfrac{P(R_1R_2R_3)}{P(R_2R_3)}$

$$P(R_1R_2R_3) = \frac{13}{52}\,\frac{12}{51}\,\frac{11}{50}$$

Now $R_2R_3 = R_2R_3R_1 \cup R_2R_3R_1^c \Rightarrow P(R_2R_3) = P(R_2R_3R_1) + P(R_2R_3R_1^c)$

$$= \frac{13}{52}\,\frac{12}{51}\,\frac{11}{50} + \frac{52-13}{52}\cdot\frac{13}{52}\cdot\frac{12}{51}$$

and $$P(R_1 \mid R_2 R_3) = \frac{\frac{13\cdot 12\cdot 11}{52\cdot 51\cdot 50}}{\frac{13\cdot 12\cdot 11}{52\cdot 51\cdot 50} + \frac{39}{52}\,\frac{13}{51}\,\frac{12}{52}} = \frac{13\cdot \cancel{12}\cdot 11}{\cancel{13\cdot 12}\cdot 11 + 39\cdot 13\cdot \cancel{12}} = \frac{11}{50} = 0.22$$

#20 Let A be the event that a student is female. We know $P(A) = 0.52$

Let B be the event that a student is majoring in CS. Then $P(B) = 0.05$

We also know that $P(AB) = 0.02$

a) $$P(A\mid B) = \frac{P(AB)}{P(B)} = \frac{0.02}{0.05} = \frac{2}{5}$$

b) $$P(B\mid A) = \frac{P(AB)}{P(A)} = \frac{0.02}{0.52} = \frac{1}{26}$$

#23 a) Let E be the event that the ball from urn II is white.

F -"- -"- I is white

$$E = EF \cup EF^c \Rightarrow P(E) = P(EF) + P(EF^c) = P(E\mid F)P(F) + P(E\mid F^c)P(F^c)$$

But $P(\text{ball from II white given ball from I white}) = \frac{2}{3} = P(E\mid F)$

Similarly
$$P(E\mid F^c) = \frac{1}{3}. \quad P(F) = \frac{1}{3}, \quad P(F^c) = 2/3$$

Thus $$P(E) = \frac{2}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3} = \frac{4}{9}$$

b) We want $$P(F\mid E) = \frac{P(EF)}{P(E)} = \frac{P(E\mid F)\,P(F)}{P(E)} = \frac{\frac{2}{3}\cdot\frac{1}{3}}{4/9} = \frac{1}{2}$$

#30 Let A the event that the selected marble is black. We want $P(A)$. Let F the event that the selected box is the one with 1 black, 1 white and the F^c -"- -"- 2 black, 1 white

$$P(A) = P(A|F)P(F) + P(A|F^c)P(F^c)$$

Black White | Black Black White

$$P(A|F) = \frac{1}{2}, \quad P(A|F^c) = \frac{2}{3}$$

$$P(F) = P(F^c) = \frac{1}{2}$$

Thus $P(A) = \frac{1}{2}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

We want $P(F|A^c) = \dfrac{P(FA^c)}{P(A^c)} = \dfrac{P(A^c|F)P(F)}{P(A^c)}$

$P(A^c|F) = \frac{1}{2}$ thus $P(F|A^c) = \dfrac{\frac{1}{2}\cdot\frac{1}{2}}{1-P(A)} = \dfrac{\frac{1}{4}}{\frac{5}{12}} = \dfrac{3}{5}$

#42 F_1 the event that the cake was baked by A. $P(F_1) = 0.5$
F_2 -"- by B $P(F_2) = 0.3$
F_3 -"- by C $P(F_3) = 0.2$

Let E be the event that the cake failed to rise.

Then $P(F_1|E) = P($given that the cake failed to rise, the event that it was baked by A$)$ = proportion of failure caused by A

But $P(E) = P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + P(E|F_3)P(F_3)$

$$= 0.02 \times 0.5 + 0.03 \cdot 0.3 + 0.05 \times 0.2 = 0.029$$

Thus $P(F_1|E) = \dfrac{P(F_1E)}{P(E)} = \dfrac{P(E|F_1)\,P(F_1)}{P(E)} = \dfrac{0.02 \cdot 0.5}{0.029} = \dfrac{10}{29}$

#43 F_1 the event that the 2 headed coin selected
F_2 the event that the fair coin selected
F_3 -11- that the biased coin selected

First calculate the probability that the coin shows head.

$$P(E) = P(E|F_1)\,P(F_1) + P(E|F_2)\,P(F_2) + P(E|F_3)\,P(F_3)$$

$$= 1 \cdot \frac{1}{3} + \frac{1}{2}\,\frac{1}{3} + \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4}$$

We want $P(F_1|E) = \dfrac{P(E|F_1)\,P(F_1)}{P(E)} = \dfrac{\frac{1}{3}}{\frac{3}{4}} = 4/9$

#51 We have $P(\text{got job} \mid \text{Strong Recommendation}) = 0.8$

$P(\text{got the job} \mid \text{Moderate Recommendation}) = 0.4$

$P(\text{got the job} \mid \text{Weak Recommendation}) = 0.1$

$P(\text{Strong Rec.}) = 0.7$, $P(\text{Moderate Rec}) = 0.2$, $P(\text{Weak Rec}) = 0.1$

a)
Thus $P(\text{got the job}) = 0.8 \times 0.7 + 0.4 \times 0.2 \times 0.1 \times 0.1 = 0.65$

b) $P(\text{Strong} \mid \text{got the job}) = \dfrac{P(\text{got the job} \mid \text{Strong})\,P(\text{Strong})}{P(\text{got the job})}$

Thus $P(\text{Strong}/\text{Got the job}) = \frac{0.8 \times 0.7}{0.65} = \frac{56}{65}$

c) $P(\text{Strong} \mid (\text{got the job})^c) = \frac{P((\text{got the job})^c \mid \text{Strong})\ P(\text{Strong})}{P((\text{got the job})^c)}$

$= \frac{0.2 \times 0.7}{1-0.65} = \frac{14}{35}$

Similarly $P(\text{Moderate} \mid J^c) = \frac{0.6 \times 0.2}{1-0.65} = \frac{12}{35}$

$P(\text{Weak} \mid J^c) = \frac{9}{35}$

#57 The daily movements are independent events. After 2 days has the same value if the first day went up and then down or vice versa.

a) Thus $P(\{\text{Up Down}, \text{Down Up}\}) = P(\text{Up Down}) + P(\text{Down Up})$

$= P(\text{Up})\,P(\text{Down}) + P(\text{Down})\,P(\text{Up}) = 2\,P(\text{Up})\,P(\text{Down}) = 2p(1-p)$

b) $P(\{\text{Up Down Up}, \text{Up Up Down}, \text{Down Up Up}\}) = 3\,P(\text{Up})P(\text{Down})$

$= 3p^2(1-p)$

c) $P(\underbrace{\text{Up on first day}}_{A} \mid \underbrace{\text{Given that in 3 days increased by 1 unit}}_{B})$

$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)} = \frac{(p(1-p)+(1-p)p) \times p}{3p^2(1-p)} = \frac{2}{3}$

since $P(B|A) = 2P(1-P)$

#64 a) Let H the event that the husband is chosen and let E the event that they get the question right.

Then $P(E) = P(E|H)P(H) + P(E|H^c)P(H^c) = P \times \frac{1}{2} + P \times \frac{1}{2} = P$

b) Let E the event that they get the question right, let H_1 the event that they agree on the correct answer, H_2 the event that they agree on the incorrect answer and H_3 the event that they disagree.

Then $P(E) = P(E|H_1)P(H_1) + P(E|H_2)P(H_2) + P(E|H_3)P(H_3) =$
$1 \times P^2 + 0 \times (1-P)^2 + \frac{1}{2} \times 2P(1-P)$

$= P^2 + P(1-P) = P.$

Thus it doesn't matter which strategy to choose.