MATH 461 : HW #6

Problem 4.45

$P(\text{pass by one examiner} \mid \text{"on" day}) = p_{on} = 0.8$, $p(\text{"on" day}) = 1/3$

$P(\text{pass by one examiner} \mid \text{"off" day}) = p_{off} = 0.4$, $p(\text{"off" day}) = 2/3$

CASE : # examiners = 3

$$P(\text{overall pass}) = \frac{1}{3}\left[\binom{3}{2} p_{on}^2 (1-p_{on}) + p_{on}^3\right] + \frac{2}{3}\left[\binom{3}{2} p_{off}^2 (1-p_{off}) + p_{off}^3\right]$$

$$= 8/15 = 5000/9375$$

CASE : # examiners = 5

$$P(\text{overall pass}) = \frac{1}{3}\left[\sum_{i=3}^{5}\binom{5}{i} p_{on}^i (1-p_{on})^{5-i}\right] + \frac{2}{3}\left[\sum_{i=3}^{5}\binom{5}{i} p_{off}^i (1-p_{off})^{5-i}\right]$$

$$= \frac{1}{3}\left[10(.8)^3(.2)^2 + 5(.8)^4(.2) + (.8)^5\right] + \frac{2}{3}\left[10(.4)^3(.6)^2 + 5(.4)^4(.6) + 0.4^5\right]$$

$$= \frac{4928}{9375}$$

$\therefore$ 3 examiners should be requested

Problem 4.48

$P(\text{1 package returned}) = P(\text{one packet "defective", 2 packets "perfect"})$

$$= \binom{3}{2} \cdot p_D \cdot (1-p_D)^2$$ where p_D = prob. that packet needs to be returned

$p_D = 1 -$ prob. (all diskettes are perfect) $-$ prob. (exactly one diskette is defective)

$$= 1 - (0.99)^{10} - \binom{10}{1}(0.99)^9(0.01) = 1 - (0.99)^9 \cdot [0.99 + 0.1]$$

$\therefore$ required probability $= 0.01268963$

Problem 4.50

$P(H) = 1 - P(T) = p$

$P(\text{6 H's in 10 flips}) = p_{6,10} = \binom{10}{6} p^6 (1-p)^4$

a.) $$P(HTT \mid \text{6 Hs in 10 flips}) = \frac{p(1-p)^2 \cdot P(\text{5 Hs in 7 flips})}{p_{6,10}} = \frac{p(1-p)^2 \binom{7}{5} p^5 (1-p)^2}{\binom{10}{6} p^6 (1-p)^4} = \frac{1}{10}$$

b.) $$P(THT \mid \text{6 Hs in 10 flips}) = \frac{(1-p)p(1-p) \cdot \binom{7}{5} p^5 (1-p)^2}{p_{6,10}} = \binom{7}{5} \Big/ \binom{10}{6} = 1/10$$

Problem 4.55 $P(\text{typist } 1) = P(\text{typist } 2) = 1/2$

$P(0 \text{ errors} \mid \text{typist } 1) = e^{-3}$, $P(0 \text{ errors} \mid \text{typist } 2) = e^{-4.2}$

$\therefore$ required probability $= \dfrac{e^{-3} + e^{-4.2}}{2}$

Problem 4.57 a.) $P(X \geq 3) = 1 - P(X < 3)$ where $X = \#$ accidents per day

$$= 1 - \left\{ e^{-3} + 3e^{-3} + \frac{9}{2} e^{-3} \right\} = 1 - \frac{17}{2} e^{-3}$$

b.) $P(X \geq 3 \mid X \geq 1) = \dfrac{P(X \geq 3)}{P(X \geq 1)} = \dfrac{1 - \frac{17}{2} e^{-3}}{1 - e^{-3}}$

Problem 4.59 a.) $P(\text{win at least once}) = 1 - P(0 \text{ wins in } 50 \text{ games})$

$$\approx 1 - e^{-50 \times 1/100} = 1 - e^{-1/2}$$

b.) $P(\text{exactly } 1 \text{ win in } 50 \text{ games}) \approx e^{-50/100} \cdot \dfrac{(50/100)^1}{1!} = \frac{1}{2} e^{-1/2}$

c.) $P(\text{at least } 2 \text{ wins in } 50 \text{ games}) = 1 - P(0 \text{ wins}) - P(1 \text{ win})$

$$\simeq 1 - e^{-1/2} - \frac{1}{2} e^{-1/2}$$

Problem 4.63 $\lambda = \frac{1}{2} \text{ min}^{-1}$, $t = 5$ min.

Let $X = \#$ persons entering between 12:00 and 12:05; Then $P(X = k) \simeq e^{-\lambda t} \cdot \dfrac{(\lambda t)^k}{k!}$

$k = 0, 1, 2, \ldots$

a.) $P(X = 0) \simeq e^{-2.5}$

b.) $P(X \geq 4) = 1 - \sum_{i=0}^{3} P(X = i) = 1 - e^{-2.5} - 2.5 e^{-2.5} - \dfrac{(2.5)^2}{2} e^{-2.5} - \dfrac{(2.5)^3}{3!} e^{-2.5}$

Problem 4.72 Let $X =$ number of games required by the stronger team to win 4 games

Then X is a negative Binomial random variable with parameters $r = 4$, $p = 0.6$

$\therefore$ $P(X = 4) = \binom{4-1}{4-1} \cdot (0.6)^4 (0.4)^{4-4} = 81/625$

$P(X = 5) = \binom{4}{3} (.6)^4 (.4) = 648/3125$ $\quad P(X = 7) = \binom{6}{3} (.6)^4 (.4)^3$

$P(X = 6) = \binom{5}{3} (.6)^4 (.4)^2 = 648/3125$ $\quad = 2592/15625$

$P(\text{team winning 2-out-of-3 series}) = p^2 + (1-p) \cdot p^2 + p(1-p) \cdot p^2 = p^2[1 + 1 - p + 1 - p] = p^2(3 - 2p)$

$$= 81/125$$

$\boxed{\text{Problem 4.78}}$ Define "success" as the event of obtaining 2 black and 2 white balls. Then

$$P(\text{"success"}) = \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = \frac{6 \times 6}{70} = \frac{18}{35}$$

Then Prob (n selections needed for success) $= P(X=n)$

where X is a Geometric Random variable with parameter $p = 18/35$

$$\Rightarrow \text{required prob.} = (1-p)^{n-1}.p = \frac{18\,(17)^{n-1}}{35^n}$$

$\boxed{\text{Problem 4.79}}$ a.)

$$\text{Prob.}\,(X=0) = \frac{\binom{94}{10}\cdot\binom{6}{0}}{\binom{100}{10}} \simeq 0.5223$$

b.)

$$P(X>2) = 1 - \sum_{i=0}^{2} P(X=i) = \frac{\binom{100}{10} - \left\{\binom{94}{10} + \binom{94}{9}\binom{6}{1} + \binom{94}{8}\binom{6}{2}\right\}}{\binom{100}{10}} \simeq 0.012($$