MATH 441 **SECTION X13** **Review Problems for the Third Exam**

Coverage: Sections 4.1-4.4, 7.1-7.6.

Problem 1 Find the solution of the given initial value problem

$$y''' + 4y' = t,\ y(0) = y'(0) = 0,\ y''(0) = 1.$$

Ans: $y = \frac{3}{16}(1 - cos(2t)) + \frac{1}{8}t^2$

Problem 2
Use the method of annihilators to find the form of a particular solution $Y(t)$ of the following differential equation

$$y''' - 2y'' + y' = t^3 + 2e^t$$

Ans: $Y(t) = t(A_0t^3 + A_1t^2 + A_2t + A_3) + Bt^2e^t$

Problem 3
Given that x, x^2, and $\frac{1}{x}$ are solutions of the homogeneous equation corresponding to

$$x^3y''' + x^2y'' - 2xy' + 2y = 2x^4,\ x > 0$$

determine a particular solution.

Ans: $Y(x) = \frac{x^4}{15}$

Problem 4 Transform the given equation into a system of first order equations

$$u^{(4)} - u = 0.$$

Ans: $\vec{x}' = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \vec{x}$

Problem 5 Verify that the given vector satisfies the given differential equation.

$$\vec{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$$

where

$$\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^t.$$

Problem 6 By using row reduction (or Gauss elimination) compute the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

Ans:

$$\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$

Problem 7 By using row reduction (or Gauss elimination) compute the solution of the following set of equations

$$\begin{gathered} x_1 + 2x_2 - x_3 = 2 \\ 2x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + 2x_3 = -1 \end{gathered}$$

Ans: $(x_1, x_2, x_3) = (-c, 1 + c, c)$, c an arbitrary constant.

Problem 8 Solve the initial value problem and describe the behavior of the solution as $t \to \infty$.

$$\vec{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \vec{x}, \ \vec{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Ans: $\vec{x} = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \frac{1}{2} \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}$

Problem 9 Solve the system of differential equations and draw the solution in the phase plane.

$$\vec{x}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{x}$$

Ans: $\vec{x} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

Problem 10 Solve the initial value problem and describe the behavior of the solution as $t \to \infty$.

$$\vec{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \vec{x}, \ \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ans: $\vec{x} = e^{-t} \cos(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t} \sin(t) \begin{pmatrix} -3 \\ -1 \end{pmatrix}$