1. **Field of study and interest:** My research has developed a framework for answering fundamental mathematical questions regarding stochastic problems. I have used this framework in real-world applications in power markets and financial risk management. My research is in the area of stochastic variational inequalities, particularly those arising from stochastic Nash games and equilibrium problems and financial risk measures. I have used techniques from convex optimization, game theory, deterministic variational inequalities, stochastic programming, probability theory, set-valued analysis and theory of financial risk measures.

More broadly, I am interested in analyzing and developing mathematical tools and techniques to model and solve practical problems to better understand the working of systems. My doctoral research and previous experience in industry, led me to explore problems at the interface of mathematics, business and economics and required an understanding of the physical/business/systemic problems, mathematical modelling and rigorous mathematical analysis to solve the problems.

2. **Research background:** Uncertainty has tremendous impact on decision making in today’s global world. The more connected we get, it seems, the more sources of uncertainty we unfold. For example, uncertainty about weather is not new; but uncertainty about prices, power, transportation, communication have stemmed from the way these networked systems operate and also how they interact with one another. Uncertainty influences the design, regulation and decisions of participants in several systems like the stock markets, electricity markets, commodity markets, wired and wireless, which are ubiquitous in today’s world. This poses many interesting mathematical questions in areas of understanding uncertainty (modelling) and dealing with uncertainty (decision making). My doctoral research focuses on answering a few fundamental mathematical questions that pertain to dealing with uncertainty. In particular my dissertation research focuses on three major topics:

   (A) Uncertainty in a game-theoretic setting 
   (B) Uncertainty in a large class of problems that are modelled as generalizations of variational inequalities ; and 
   (C) Uncertainty as seen in the way risk is measured in financial industry (using Value at Risk) and its major consequences to the financial health of a firm.

The mathematical theory of Nash games and variational inequalities have seen a lot of practical relevance in industry and business settings because they are natural models for many real-world applications such as frictional contact problems, traffic equilibrium problems, pricing and congestion in communication networks, Nash-Cournot production-distribution games in power markets, market-based allocation of resources and so on. Incorporating uncertainty into Nash games and variational inequalities leads us to **stochastic Nash games** and **stochastic variational inequalities** respectively.

3. **Research Summary:** My research pertaining to (A), (B) and (C) above is summarized next. More detailed contributions are presented in section 4 and plans for future research is in section 5.

(A), (B) **Stochastic Nash games and variational inequalities:** My doctoral research has focused on developing a novel framework for analyzing stochastic variational inequalities on a continuous probability space and their applications. A framework is provided for the tractable verification of existence of solutions to such problems. Though the theory for deterministic analogues is rich, the theoretical properties of such problems in stochastic regimes are less understood. The main challenge in directly applying deterministic results to stochastic regimes is that such an approach involves the evaluation of the expectation and its derivatives, a highly nonlinear operator. The feature of this research that sets it apart from other endeavors is that the proposed framework does
not necessitate the evaluation of an expectation or its derivatives, which in practical settings is a huge advantage.

Applications areas of these problems include extensions of classical (and generalized) Nash games to stochastic regimes as well as more general stochastic equilibrium problems, such as traffic equilibrium problems and economic equilibrium problems under uncertainty. In practical settings, such games are frequently seen in pricing and congestion in communication networks, games in power networks, market-based allocation of resources and production-distribution Nash-Cournot games.

(C) Risk management:

In a financial firm, what can we say about a managers’ ability to manage a traders’ risk?

I have developed a model to help answer this question and a mathematical proof for a conjecture that in certain settings (distributions with unbounded support such as Gaussian, exponential or fat-tailed distributions), a financial firm that manages risk using the industry standard VaR (Value-At-Risk) risk measure, actually gives its risk-seeking traders the freedom to take on an extremely high amount of CVaR (Conditional VaR) risk, thereby allowing for the firms’ ruin. This work also has practical applications to risk management as practiced today in the financial industry.

4. Research contributions:

(A) Stochastic Nash games and generalizations: This research is motivated by the need to characterize the solution sets of stochastic Nash games particularly when such games are characterized by expected-value objectives, nonsmoothness and stochastic constraints. By leveraging Lebesgue convergence theorems, variational analysis and the theory of set-valued integrals, I provide a framework for tractable sufficiency conditions for the existence of a stochastic Nash equilibrium that only involves the analysis of the integrands of the expectation. Importantly, these conditions do not necessitate the evaluation of an expectation or its derivatives. In particular, I consider the related scenario-based Nash game which is the deterministic Nash game corresponding to the realization of a particular scenario of the uncertainty. I provide sufficient conditions which when satisfied by the scenario-based Nash game in an almost-sure sense guarantee existence of solution to the original stochastic Nash game. The specific contributions may be characterized as follows:

(i) Smooth stochastic Nash games: Under suitable smoothness assumptions and when strategy sets are possibly coupled through a shared convex constraint, I show that when the scenario-based Nash game satisfies a coercivity property in an almost-sure sense, the smooth stochastic Nash game admits an equilibrium. Notably, this coercivity condition also guarantees the existence of equilibrium of the scenario-based Nash game in an almost-sure sense. However, for uniqueness of stochastic equilibrium, we require the scenario-based Nash game to satisfy a monotone property in an almost-sure sense and a strict monotonicity property only on a set of positive measure in order to claim uniqueness of equilibrium for the stochastic Nash game.

(ii) Nonsmooth stochastic Nash games: When player payoffs are merely continuous, the variational conditions of corresponding scenario-based Nash games are given by multivalued variational inequalities. By utilizing a set-valued analog of Fatou’s Lemma and Aubin’s theory of set-valued integrals, the existence relationship of (i) may be recovered for shared-constraint stochastic Nash games.

(iii) Stochastic Nash games with coupled stochastic constraints: When strategy sets are coupled by shared convex expected-value constraints, a suitable regularity condition allows for claiming existence and uniqueness in the primal-dual space.

1The research in this area was recently recognized at the triennial International Conference on Stochastic Programming (2010) as the most outstanding student-authored paper and the citation mentioned that the research has opened a new realms of applications in computational game theory.

2This research in stochastic Nash games has resulted in a manuscript “On the Characterization of Solution Sets of Smooth and nonsmooth Stochastic Nash Games.” which has recently been published in SIAM Journal of Optimization, 2011. A shorter version of the paper appears in the conference proceedings: “Proceedings of the American Control Conference (ACC), Baltimore, 2010.”
(iv) **Examples:** Finally, I illustrate the framework by examining two extensions of stochastic Nash-Cournot games, of which the first allows for nonsmooth payoffs through the introduction of risk-measures while the second allows for shared stochastic constraints.

**(B) Stochastic variational inequalities and generalizations:** The second part of my research extends the above framework to study stochastic variational inequalities and their generalizations and stochastic complementarity problems. Direct application of deterministic results for QVIs and CPs to stochastic regimes is difficult unless a tractable form of the expectation and derivatives is available. When smoothness requirements are relaxed on the integrands of the expected value objectives, even less may be said about the existence of solution to such problems. My research provides amongst the first attempts to examine this class of stochastic problems using an approach similar to the above framework. The contributions can be summarized as:

(i) **General framework:** We provide sufficient conditions for the scenario-based analogue of the stochastic problem. When these conditions are satisfied in an almost-sure sense by the scenario-based problem, the existence of solution to the corresponding stochastic problem is guaranteed.

(ii) **Stochastic quasi-variational inequalities and generalizations:** The sufficient conditions in such stochastic problems QVI$(K, F)$ are given in terms of continuity properties of the set-valued map $K$ and the boundedness of an appropriate ‘level set’ of the smooth function $F$. Another sufficient condition is provided in terms of almost-sure coercivity of the scenario-based map of $F$ and continuity of the map $K$. Existence under compactness of set-valued map $K$ is proved as a corollary to a result by Harker. Using Aubin’s results for set-valued integrals, I am able to derive similar sufficient conditions for the scenario-based generalized quasi-variational inequality problem which when satisfied in an almost-sure sense guarantees the existence of solution of the corresponding stochastic problem.

(iii) **Stochastic complementarity problems:** For such problems I am able to show that if a co-coercivity property holds in an almost-sure sense for the functions of the scenario-based problems then existence of solution to the stochastic problems may be claimed.

(iv) **Applications to Nash games and traffic equilibrium problems:** We show how our framework can be used to claim existence of a generalized stochastic Nash equilibrium in a stochastic Nash-Cournot electricity pricing model. The framework is also illustrated in the study of equilibrium properties of stochastic traffic equilibrium problems.

A journal version of this paper is currently being completed and is almost ready for submission.

**(C) Risk measures.** Risk measures are functions that capture the riskiness of an asset like a portfolio. Risk measures typically allocate a number to a risky asset, thereby allowing for comparison of various risky assets held by a firm. There are two types of risk measures predominantly used in financial risk management. Widely accepted by the financial industry and regulators alike and extensively used by practitioners, the value at risk (VaR) is essentially a measurement of quantiles. However, the academic field of mathematical finance has long realized that VaR has certain noticeable shortcomings; for example, it does not properly capture diversification. Alternate measures have been suggested in the form of coherent risk measures which do, in fact, have many desirable properties. One such coherent measure is Conditional Value-at-Risk, or CVaR which is often more tractable in optimization applications. While the choice of risk measures is a crucial question in the context of risk management, the focus of this research is how these measures interact.

**Motivating problem and contributions.** Consider a typical day at $\infty$—Alpha Asset Management, a fictitious risk management firm, where Mike manages a trader, Theresa. Mike’s training is in classical finance while Theresa has a Ph.D. in mathematics. Mike imposes a VaR constraint on Theresa’s trades. Theresa’s mathematically-trained brain works on a mathematical understanding of risk, and she trades on a coherent risk measure. Since she is seeking higher returns, she maximizes her risk; however, she is constrained by Mike’s VaR budget. In other words, she will trade so as to maximize her coherent risk measure subject to Mike’s VaR constraints.
What can we learn about Mike’s ability to manage Theresa’s risk?

To answer the question, the problem is modeled as a VaR constrained CVaR maximization problem and is referred to as the Trader’s problem. The presence of the VaR constraint makes the problem nonconvex. The problem reduces to obtaining a global solution of a stochastic nonconvex program, which is generally challenging. I have shown that for general distributions with unbounded support, such as Gaussian distributions, exponential distributions and fat-tailed distributions, the global optimal value of the Trader’s problem is unbounded (i.e an unbounded amount of CVaR risk can be sought). I also show that for general distributions with bounded support, such as uniform distributions, the maximum possible CVaR risk remains bounded. As a result, I am able to analytically show that in certain settings, it is possible for a trader to take on a huge amount of CVaR risk while maintaining the VaR threshold imposed by the firm to manage risk. This shows that while a VaR threshold is a tool used to manage risk at large banks etc, in certain settings it also gives the traders the freedom to undertake an extremely high amount of CVaR risk. This could lead the financial firms to ruin. It calls for better risk management tools to be used in practice to ensure that an unbounded amount of CVaR risk is not allowed. A manuscript describing this research “VAR, CVaR, and nonlinear interactions.” has recently been submitted to Operations Research.

In a 2005 paper, “Has Financial Development Made the World Riskier?” Raghuram Rajan has argued that investment firm managers are given incentives to take risk that generate severe adverse consequences with small probability but, in return, offer generous compensation the rest of the time. I believe that there is serious concern about such problems but no simple models exist to show the true impact. The Trader’s problem described above seems to be a simple model in this direction.

5. Future directions: In the coming years, while continuing to extend my doctoral research, I would explore a few more questions which I believe can be answered by appropriate modifications to the framework I developed. For example,

1. The question of the stability of stochastic problems under perturbations of the probability measure itself as well as under perturbations of the underlying data is an interesting, challenging, important and a practically relevant problem. Once again, the non-linearity of the expectation has proved to be a challenge in addressing this problem. However, this problem seems amenable to being tackled using a modification of the framework I have developed during my doctoral research.

2. Another extension of my prior work would be to relax the convexity assumption. I am keen on using the same framework with a fixed point approach instead of a variational one to yield equilibria.

3. In the area of financial risk management, my work has close relation to risk preferences and incentives in the principal-agent problem in economics. I am keen on exploring this aspect further with faculty in mathematical finance and economics.

My doctoral research and previous experience in the industry has led me to explore problems at the interface of mathematics, business and economics. This together with my background in the private sector will provide topics for fruitful interdisciplinary projects as well as industry collaborations at undergraduate and graduate level. I am also interested in using my industry experience to develop industry partnerships for math-in-industry type projects. I am keen and well positioned to conduct research in emerging areas such as energy markets, systemic risk attribution, operations research and health care systems and role of incentives in the principal-agent problem; all of which require a firm grounding in theory, applications and interdisciplinary research. As mentioned at the beginning, at a very broad level, as a researcher, I am interested in analyzing and developing mathematical tools and techniques to model and solve practical problems to better understand the working of systems.

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