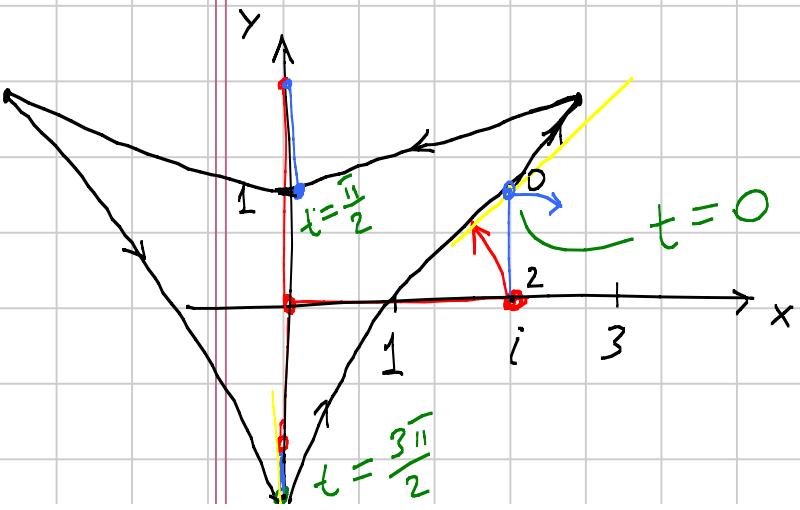
Calculus and parametric equations

Note Title 4/13/2009

Scrambler



$x_i = 2\cos t$, i-internal

$y_i = 2\sin t$

$x_o = \sin 2t$

$y_o = \cos 2t$

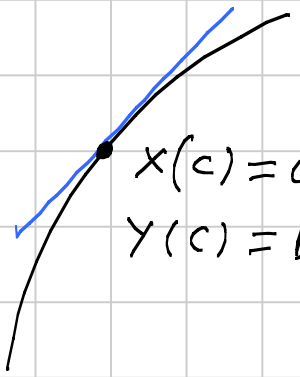
$t=\frac{3\pi}{2}$, $y_i = 2\sin\left(\frac{3\pi}{2}\right) =$

$= -2$

$$y_0 = \cos\left(2 \cdot \frac{3\pi}{2}\right) =$$

$$= \cos(3\pi) = -1$$

Slopes of tangent lines



What is the slope of tangent line through (a, b)

$$y = y(x(t))$$

$$\frac{dy}{dt}(c) = \frac{dy}{dx}(a) \cdot \frac{dx}{dt}(c) \quad \Rightarrow$$

$$\frac{dy}{dx}(a) = \frac{\frac{dy}{dt}(c)}{\frac{dx}{dt}(c)}$$

Example 1

$x = 3t$

$y = (3t)^2$

$y = x^2$ is the corresponding x-y equation

Find $\frac{dy}{dx}$ at $t = c$ (it corresponds to $x = 3 \cdot c = a$)

$$\frac{dy}{dx} = y'_x = 2x, \quad y'_x(a) = 2a = 6 \cdot c$$

Now, use our formula

$$\frac{dy}{dx}(a) = \frac{\frac{dy}{dt}(c)}{\frac{dx}{dt}(c)} = \frac{((3t)^2)'}{(3t)'}\Big|_{t=c} = \frac{9 \cdot 2t}{3}\Big|_{t=c} =$$

$= 6 \cdot t \big|_{t=c} = \underline{6 \cdot c}$ the same answer.

Slope of the scrambler curve

Recall: $x(t) = x_i(t) + x_o(t) = 2\cos t + \sin 2t$

$y(t) = y_i(t) + y_o(t) = 2\sin t + \cos 2t$

$x'(t) = -2\sin t + 2\cos 2t$

$y'(t) = 2\cos t - 2\sin 2t$

At $t=0$ $$\left.\frac{dY}{dx}\right|_{t=0} = \frac{Y_t'(0)}{X_t'(0)} = \frac{2}{2} = 1$$

At $t=\frac{3\pi}{2}$ $$\left.\frac{dY}{dx}\right|_{t=\frac{3\pi}{2}} = \frac{Y_t'\left(\frac{3\pi}{2}\right)}{X_t'\left(\frac{3\pi}{2}\right)} = \frac{0}{0} \; ?$$

Consider the limit $t \to \frac{3\pi}{2}$

Use l'Hopital's rule

$$\lim_{t \to \frac{3\pi}{2}} \frac{y'(t)}{x'(t)} = \lim_{t \to \frac{3\pi}{2}} \frac{y''(t)}{x''(t)} = \frac{6}{0} = \infty$$

tangent is vertical