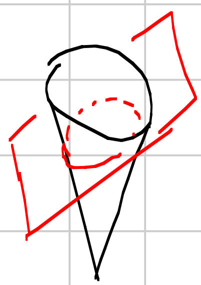
Conic Sections

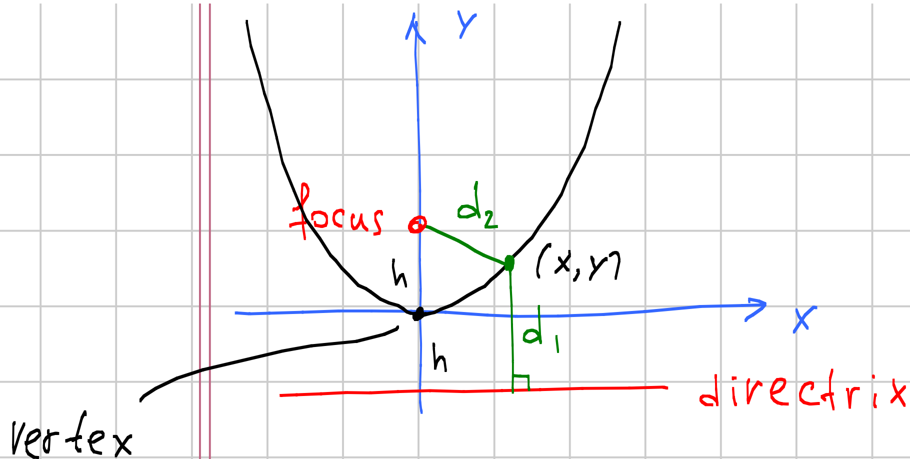
Note Title 5/4/2009



Conic sections are obtained by intersecting plane with a cone.

Parabolas

Parabola is a set of points, which have the same distance from a focus and a directrix.



$$d_1 = d_2 \quad , \qquad d_1 = y + h$$

$$d_2 = \sqrt{x^2 + (y-h)^2}$$

$$d_1^2 = d_2^2$$

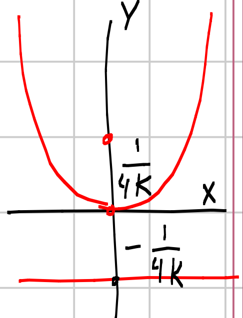
$$(y+h)^2 = x^2 + (y-h)^2$$

$$\cancel{x^2} + 2hy + \cancel{h^2} = x^2 + \cancel{y^2} - 2hy + \cancel{h^2}$$

$$4hy = x^2 \Rightarrow y = \frac{1}{4h} x^2$$

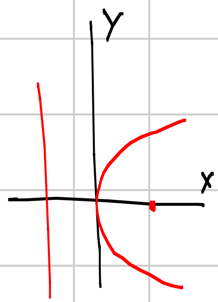
<u>Important property</u>:

Find the focus of a given parabola



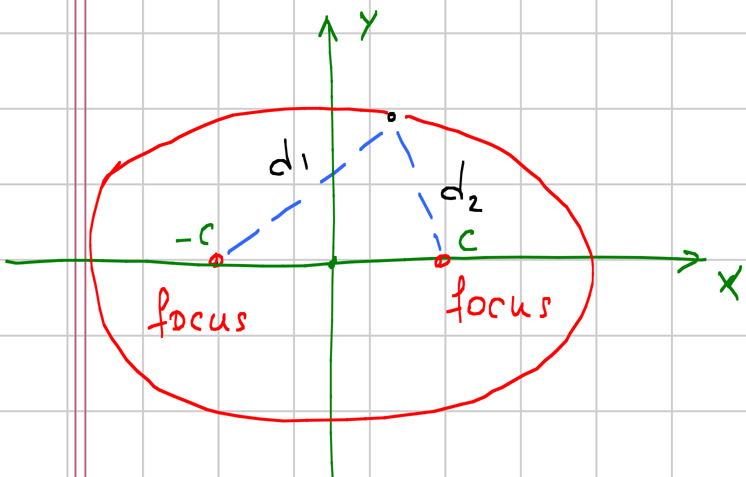
$y = k x^2$, **Recall**: $y = \frac{1}{4h} x^2$,

wher h is distance between focus and vertex (or half the distance between focus and directrix).



$h = \frac{1}{4K}$, e.g. $y = x^2$, $h = \frac{1}{4}$

Ellipse: The set of points for which $d_1 + d_2 = \text{constant} = K > 2c$



$2c$ = distance between the foci.

$$d_1 = \sqrt{(x+c)^2 + y^2}$$

$$d_2 = \sqrt{(x-c)^2 + y^2}$$

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = K$$

$$\sqrt{(x+c)^2+y^2} = k - \sqrt{(x-c)^2+y^2}$$

$$(x+c)^2+y^2 = k^2 + (x-c)^2 + y^2 - 2k\sqrt{(x-c)^2+y^2}$$

$$\cancel{x^2}+2cx+\cancel{c^2}+\cancel{y^2} = k^2 + \cancel{x^2} - 2cx + \cancel{c^2} + \cancel{y^2} - 2k\sqrt{(x-c)^2+y^2}$$

$$4cx - k^2 = -2k\sqrt{(x-c)^2+y^2}$$

$$(4cx-k^2)^2 = 4k^2\left((x-c)^2+y^2\right)$$

$$16c^2x^2 \underline{-8ck^2x} + k^4 = 4k^2(x^2 \underline{-2cx} + c^2 + y^2)$$

Let $k=2a$

$b^2=a^2-c^2>0$

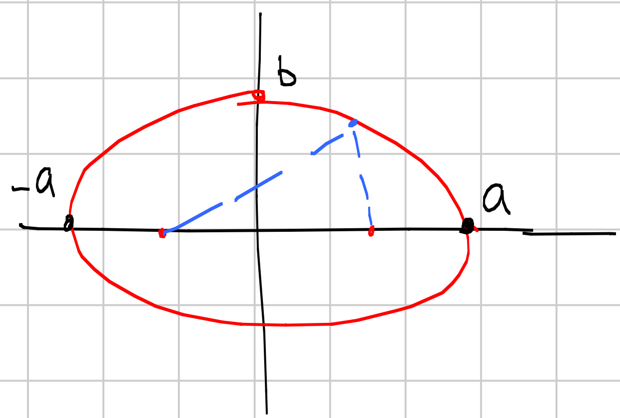
$$(4k^2-16c^2)x^2 + 4k^2y^2 = k^4 - 4k^2c^2$$

$$\cancel{16}(a^2-c^2)x^2 + \cancel{16}a^2y^2 = \cancel{16}a^2(a^2-c^2)$$

$b^2x^2 + a^2y^2 = a^2b^2$, divide by a^2b^2

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

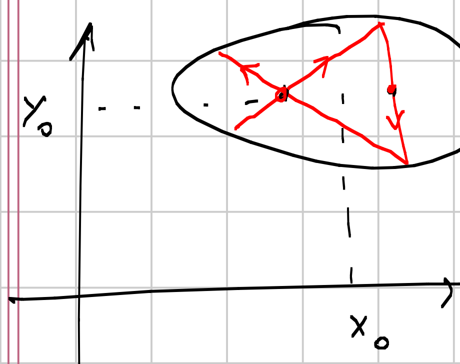
Note, $a^2 - c^2 = \left(\frac{k}{2}\right)^2 - c^2 > 0$, since $k > 2c$



If $y = 0$, then $x = a$

$(a,0)$ and $(-a,0)$ are vertices

If $a > b$, then $2a$ is major axis
$2b$ is minor axis



$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$