

Parametric Equations

Note Title

4/8/2009

Example 1: $x = t^2 - 2$

$$y = t^3 - t$$

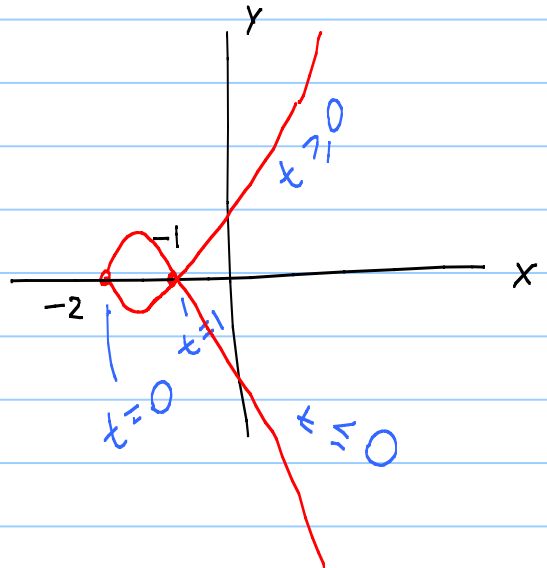
t - arbitrary

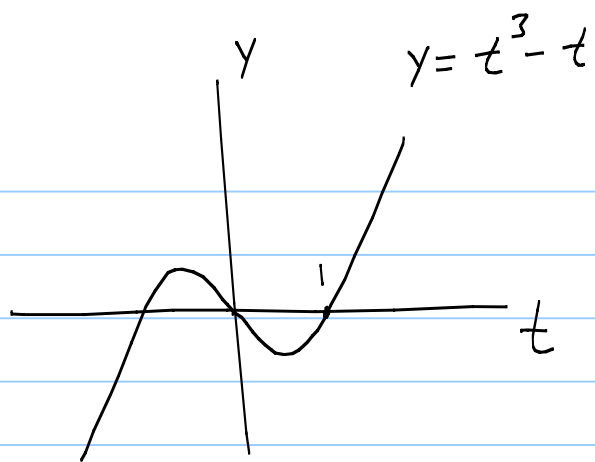
Symmetry: $t \rightarrow -t$

$$(x, y) \rightarrow (x, -y)$$

$$t \geq 0; \quad t = 0, \quad (x, y) = (-2, 0)$$

$t \nearrow$ then $x \nearrow$





Note

$$t = 1$$
$$y(1) = 0$$
$$x(1) = -1$$

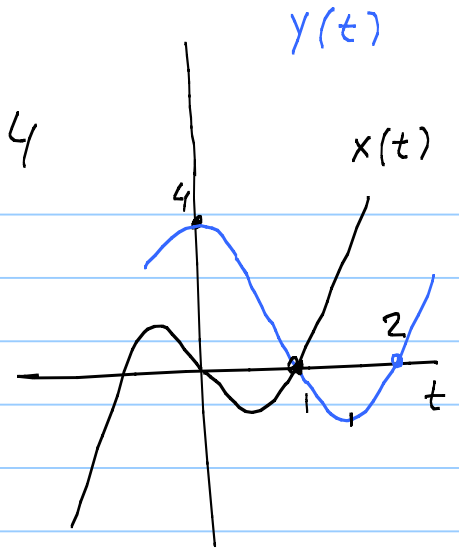
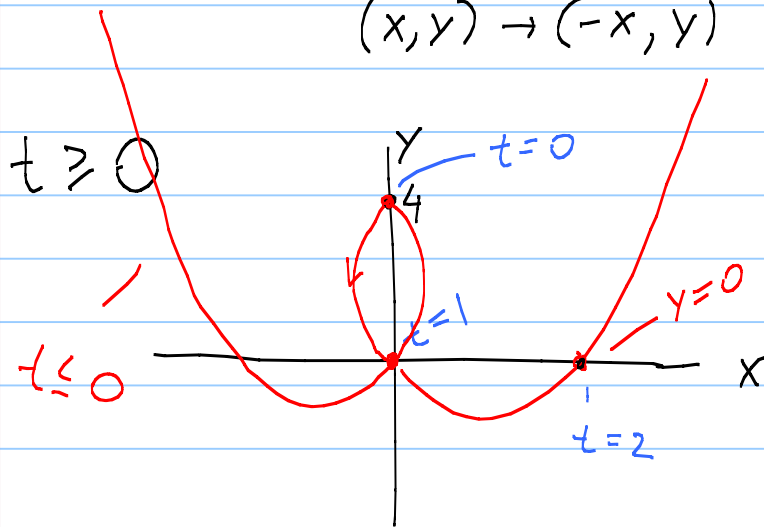
Example 2

$$x = t^3 - t$$

$$y = t^4 - 5t^2 + 4$$

Symmetry: $t \rightarrow -t$

$$(x, y) \rightarrow (-x, y)$$



$$x = 0 \text{ if } t^3 - t = 0$$

$$t(t-1)(t+1) = 0$$

$$t = 1, x = 0, y = 0$$

$$y=0 \Rightarrow t^4 - 5t^2 + 4 = 0 \quad t^2 = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} = 4, 1$$

Hence, $y=0$ if $t=1, 2$

(also $y=0$ if $t=-1, -2$)

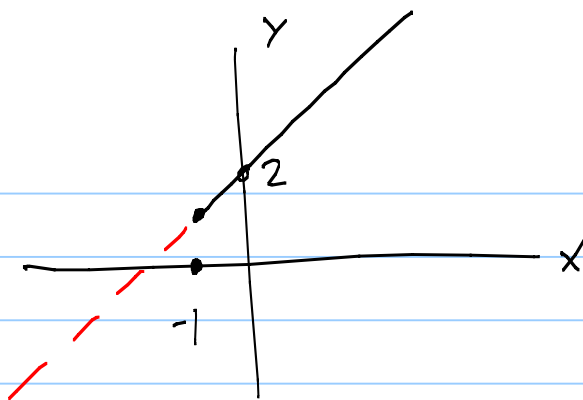
Example 3

$$x = t^2 - 1$$

$$y = t^2 + 1$$

Get $x-y$ equation
and sketch the curve

$$y - x = 2, \quad x \geq -1$$

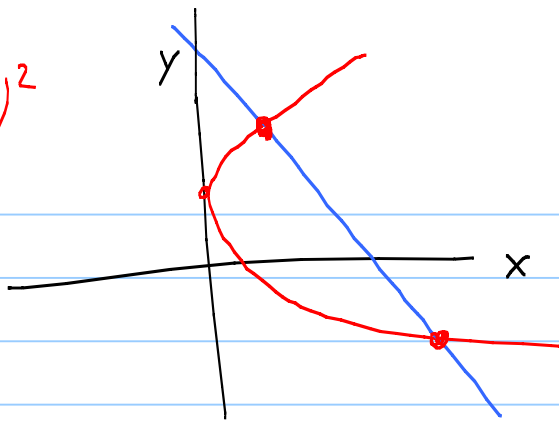


Note: $x = t^2 - 1 \geq -1$

Example 4 Intersection of 2 curves

$$\begin{array}{l}
 X = t^2 \quad \bullet \quad X = 2 + s \\
 Y = t + 1 \quad \bullet \quad Y = 1 - s
 \end{array}
 \rightarrow x + y = 3, y = 3 - x$$

$$x = (y-1)^2$$



$$t = 1, (x, y) = (1, 2)$$

$$t = -2, (x, y) = (4, -1)$$

$$y: t + 1 = 1 - s$$

$$x: t^2 = 2 + s$$

$$s + t = 0$$

$$t^2 = 2 - t$$

$$t^2 + t - 2 = 0$$

$$(t - 1)(t + 2) = 0$$

$$t = 1, s = -1$$

$$t = -2, s = 2$$