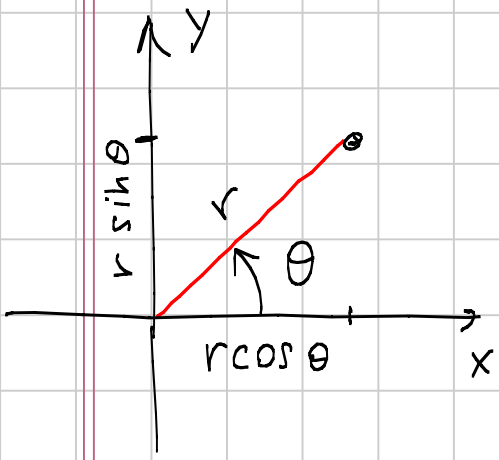
Polar Coordinates

Note Title 4/22/2009



(x,y) - rectangular coordinates

(r,θ) - polar coordinates

$r \geq 0$, $-\infty < \theta < \infty$

$(r,\theta) \sim (r, \theta \pm 2\pi)$

$$(*) \quad \begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

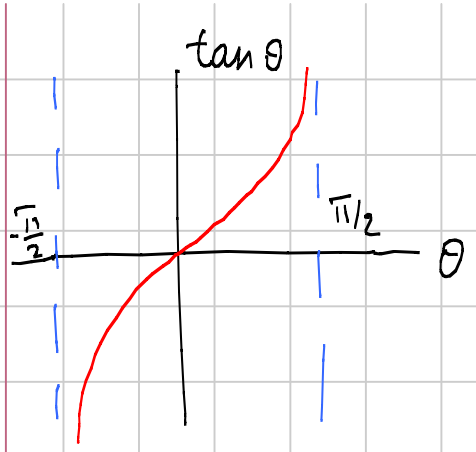
To convert from polar to rectangular coordinates, use (*).

To convert from rectangular to polar coordinates,

$$x^2+y^2 = r^2(\cos^2\theta+\sin^2\theta)$$

$$x^2+y^2 = r^2, \quad r = \sqrt{x^2+y^2}$$

$$\frac{y}{x} = \tan\theta$$

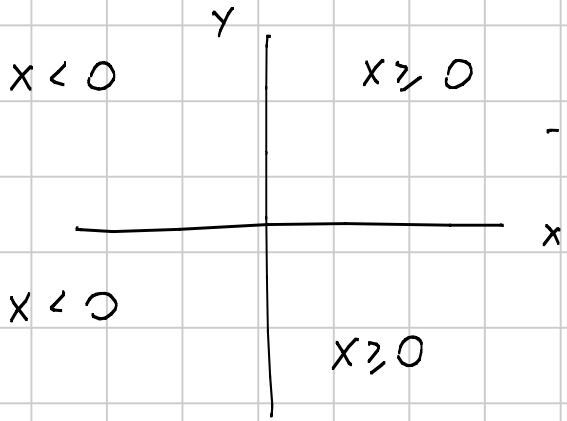


If $x \geq 0$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

If $x < 0$, then

$\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$

$\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

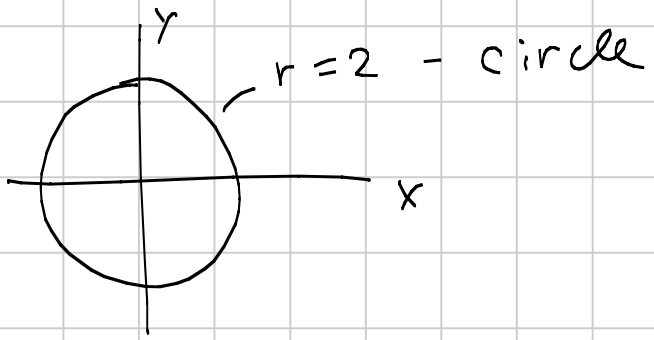


$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

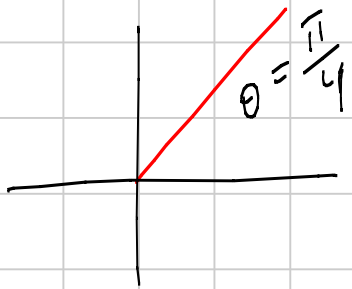
Note: In the text, the range of r is defined to be $-\infty < r < \infty$.

Graphs in polar coordinates

(1) $r = 2$



(2) $\theta = \frac{\pi}{4}$

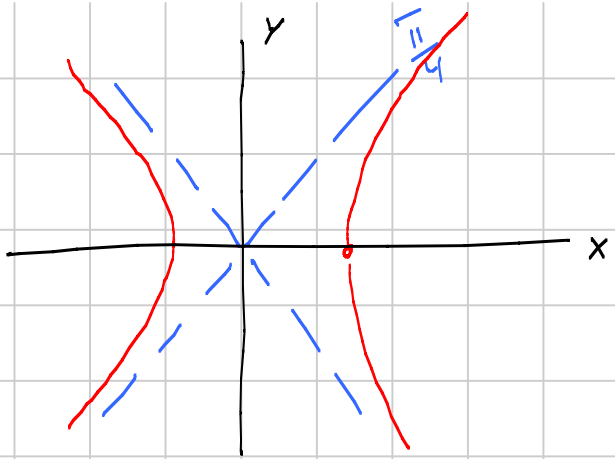


(3) Hyperbola, $x^2 - y^2 = 1$

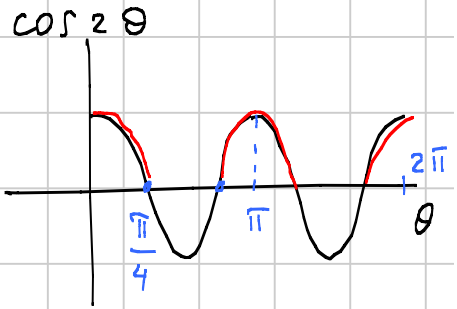
In polar coordinates, $r^2(\cos^2\theta - \sin^2\theta) = 1$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

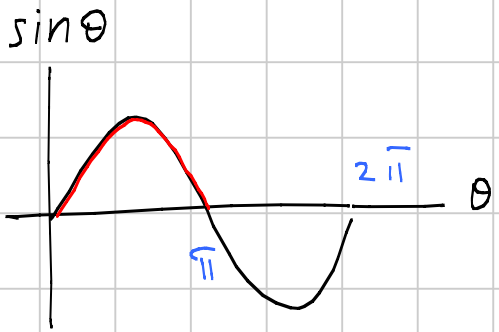
$$r^2 = \frac{1}{\cos 2\theta}$$



Need $\cos 2\theta > 0$



(4) $r = \sin\theta$



Looks like a circle: verify!

Multiply by r: $r^2 = r\sin\theta$

Equivalent to

$$x^2 + y^2 = y$$

$$x^2 + y^2 - y = 0$$

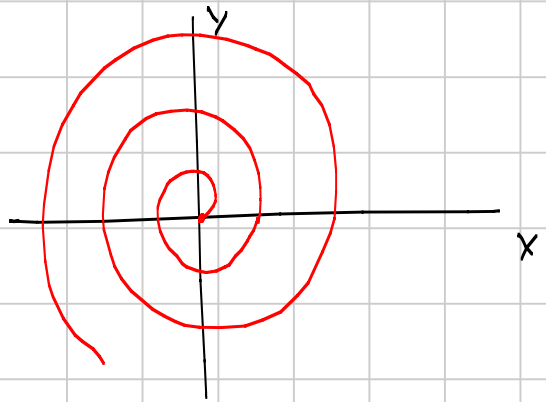
$$x^2 + \left(y - \tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2}\right)^2$$ – this is eqn.

of a circle of radius $\frac{1}{2}$ centered at $y = \frac{1}{2}$

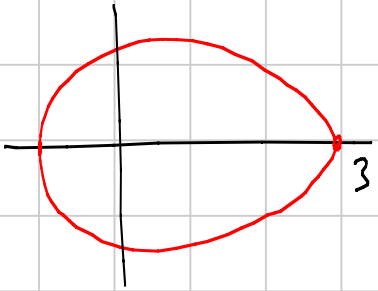
(5) Archimedian Spiral $r = \theta$

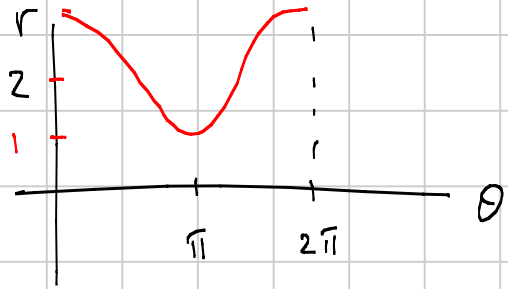
Note in rectangular coordinates

$$\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{y}{x}\right).$$

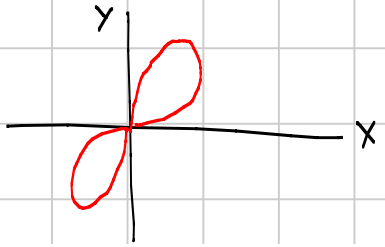


(6) Limaçon $r = 2 + \cos\theta$





(7) $r = \sin 2\theta$



$r = \sin 4\theta$