MATH 012 Test 1 Key

1. Complete the statement of the quadratic formula: If $ax^2 + bx + c = 0$ and $a \neq 0$, then:

$$x =$$

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Solve $2t^2 - 8t = 30$ for t by completing the square.

Solution:

$$\begin{aligned}
2t^2 - 8t &= 30 \\
\frac{2}{2}t^2 - \frac{8}{2}t &= \frac{30}{2} \\
t^2 - 4t &= 15 \\
t^2 - 4t + 4 &= 19 \text{ since } \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4 \\
(t-2)^2 &= 19 \\
t - 2 &= \pm\sqrt{19} \\
t &= 2 \pm \sqrt{19}
\end{aligned}$$

3. Solve $\sqrt{3n-5} - \sqrt{n-1} = 2$ for n.

Solution:

$$\begin{aligned}
\sqrt{3n-5} - \sqrt{n-1} &= 2 \\
\sqrt{3n-5} &= 2 + \sqrt{n-1} \\
(\sqrt{3n-5})^2 &= (2 + \sqrt{n-1})^2 \\
3n - 5 &= 4 + 4\sqrt{n-1} + n - 1 \\
3n - 5 &= n + 3 + 4\sqrt{n-1} \\
2n - 8 &= 4\sqrt{n-1} \\
n - 4 &= 2\sqrt{n-1} \\
(n-4)^2 &= (2\sqrt{n-1})^2 \\
n^2 - 8n + 16 &= 4(n-1) \\
n^2 - 8n + 16 &= 4n - 4 \\
n^2 - 12n + 20 &= 0 \\
(n-10)(n-2) &= 0
\end{aligned}$$

$$n - 10 = 0 \quad \text{or} \quad n - 2 = 0$$
$$n = 10 \quad \text{or} \quad n = 2$$

Since $\sqrt{3 \cdot 10 - 5} - \sqrt{10 - 1} = \sqrt{25} - \sqrt{9} = 2$ and $\sqrt{3 \cdot 2 - 5} - \sqrt{2 - 1} = \sqrt{1} - \sqrt{1} = 0$, then $n = 10$ is the only solution.

4. Simplify $\sqrt{32s^2t^3u^4}$.

Solution: $\sqrt{32s^2t^3u^4} = \sqrt{16s^2t^2u^4}\sqrt{2t} = 4|s|tu^2\sqrt{2t}$

5. Simplify $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}$.

Solution: $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{\left(\frac{1}{x}-\frac{1}{y}\right)xy}{\left(\frac{1}{x}+\frac{1}{y}\right)xy} = \dfrac{\frac{xy}{x}-\frac{xy}{y}}{\frac{xy}{x}+\frac{xy}{y}} = \dfrac{y-x}{y+x}$

6. Simplify $\left(\dfrac{8x^4y^6}{135z^8}\right)^{\frac{-1}{3}}$.

Solution:
$$\begin{aligned}\left(\frac{8x^4y^6}{135z^8}\right)^{\frac{-1}{3}} &= \left(\frac{5\cdot 27z^8}{8x^4y^6}\right)^{\frac{1}{3}}\\ &= \left(\frac{27z^6}{8x^3y^6}\right)^{\frac{1}{3}}\left(\frac{5z^2}{x}\right)^{\frac{1}{3}}\\ &= \frac{3z^2}{2xy^2}\left(\frac{5x^2z^2}{x^3}\right)^{\frac{1}{3}}\\ &= \frac{3z^2\sqrt[3]{5x^2z^2}}{2x^2y^2}\end{aligned}$$

7. If $A=\{0,1,2,3,4\}$ and $B=(1,3]$, what is $A\cap B$?

Solution: The only elements of A that also belong to B are 2 and 3. Thus, $A\cap B=\{2,3\}$.

Note that $\{0,1,2,3,4\}$ is *not* the same as $[0,4]$.
Note also that 1 is *not* an element of $(1,3]$.

8. Determine the distance between $\sqrt{2}$ and $\sqrt{5}$.

Solution: $|\sqrt{2}-\sqrt{5}|=\sqrt{5}-\sqrt{2}$ (Remember not to round.)

9. Convert 6,050,000 to scientific notation.

Solution: $6,050,000 = 6.05\cdot 10^6$

10. Convert 0.0086706 to scientific notation.

Solution: $0.0086706 = 8.6706\cdot 10^{-3}$

11. Convert $5.2\cdot 10^4$ to decimal notation.

Solution: $5.2\cdot 10^4 = 52,000$

12. Convert $8.27\cdot 10^{-5}$ to decimal notation.

Solution: $8.27\cdot 10^{-5} = 0.0000827$

13. Rationalize the numerator of $\dfrac{\sqrt{5}+\sqrt{2}}{6}$.

Solution:

$$\frac{\sqrt{5}+\sqrt{2}}{6} = \frac{\sqrt{5}+\sqrt{2}}{6} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{5-2}{6(\sqrt{5}-\sqrt{2})} = \frac{3}{6(\sqrt{5}-\sqrt{2})} = \frac{1}{2(\sqrt{5}-\sqrt{2})}$$

Remember to reduce all fractions whenever possible.

14. Factor $5x^6 - 5y^6$.

Solution:
$$\begin{aligned} 5x^6 - 5y^6 &= 5(x^6 - y^6) \\ &= 5(x^3 + y^3)(x^3 - y^3) \\ &= 5(x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2) \end{aligned}$$

Remember that "factor" always means "factor completely".

15. Solve $3|t-8| + 7 = 5$ for t.

Solution:

$$\begin{aligned} 3|t-8| + 7 &= 5 \\ 3|t-8| &= -2 \\ |t-8| &= \frac{-2}{3} \end{aligned}$$

$$\emptyset$$

16. Determine the y-intercept of $3x - 2y = \sqrt{2}$.

Solution:

$$\begin{aligned} 3x - 2y &= \sqrt{2} \\ -2y &= -3x + \sqrt{2} \\ y &= \frac{-3}{-2}x + \frac{\sqrt{2}}{2} \\ y &= \frac{3}{2}x - \frac{\sqrt{2}}{2} \end{aligned}$$

The y-intercept is at $\left(0, \dfrac{-\sqrt{2}}{2}\right)$.

17. Determine the equation in slope-intercept form of the line that is perpendicular to $6x - 2y = 7$ and passes through the point $(1, 1)$.

Solution:

$$\begin{aligned} 6x - 2y &= 7 \\ -2y &= -6x + 7 \\ y &= \frac{-6}{-2}x + \frac{7}{-2} \\ y &= 3x - \frac{7}{2} \end{aligned}$$

The desired line has slope $\frac{-1}{3}$ (the opposite reciprocal of 3) and passes through the point $(1, 1)$. This information should be plugged into point-slope form.

$$\begin{aligned} y - 1 &= \frac{-1}{3}(x - 1) \\ y - 1 &= \frac{-1}{3}x + \frac{1}{3} \\ y &= \frac{-1}{3}x + \frac{4}{3} \end{aligned}$$

18. Determine the domain of $g(x) = \dfrac{\sqrt{x+8}}{\sqrt{3-x}}$.

Solution: It must be the case that $x + 8 \geq 0$ and $3 - x > 0$. Since $x + 8 \geq 0$, then $x \geq -8$. Since $3 - x > 0$, then $x < 3$. Thus, $-8 \leq x < 3$.

19. If $f(x) = x^2 - 3$, determine and simplify $\dfrac{f(x+h) - f(x)}{h}$.

Solution: Note that $f(x + h) = (x + h)^2 - 3 = x^2 + 2hx + h^2 - 3$. Thus,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{x^2 + 2hx + h^2 - 3 - x^2 + 3}{h} \\ &= \frac{2hx + h^2}{h} \\ &= \frac{h(2x + h)}{h} \\ &= 2x + h. \end{aligned}$$

20. Solve $|3x+8| \le 7$ for x.

Solution:

$$\begin{aligned} 3x+8 &\le 7 && \text{and} & 3x+8 &\ge -7 \\ 3x &\le -1 && \text{and} & 3x &\ge -15 \\ x &\le \frac{-1}{3} && \text{and} & x &\ge -5 \end{aligned}$$

$$-5 \le x \le \frac{-1}{3}$$

Note that both $\left[-5, \frac{-1}{3}\right]$ and a properly shaded number line were also acceptable answers.

21. Solve $\frac{1}{x} - \frac{x+10}{x+1} \ge -5$ for x.

Solution:

$$\frac{1}{x} - \frac{x+10}{x+1} \ge -5$$

$$\begin{aligned} \frac{x+1}{x(x+1)} + \frac{-x(x+10)}{x(x+1)} + 5 &\ge 0 \\ \frac{x+1-x^2-10x+5x(x+1)}{x(x+1)} &\ge 0 \\ \frac{-x^2-9x+1+5x^2+5x}{x(x+1)} &\ge 0 \\ \frac{4x^2-4x+1}{x(x+1)} &\ge 0 \\ \frac{(2x-1)^2}{x(x+1)} &\ge 0 \end{aligned}$$

$(2x-1)^2$	+	+	+	+
x	−	−	+	+
$x+1$	−	+	+	+
$\frac{(2x-1)^2}{x(x+1)}$	+	−	+	+
	-1	0	$\frac{1}{2}$	

Thus, the answer is $(-\infty,-1) \cup (0,\infty)$.

22. Convert $0.8\overline{45}$ to a fraction. Be sure to show all work.

Solution:

$$\begin{aligned}
x &= 0.8\overline{45} \\
100x &= 84.5\overline{45} \\
100x - x &= 84.5\overline{45} - 0.8\overline{45} \\
99x &= 83.7 \\
990x &= 837 \\
x &= \frac{837}{990} \\
x &= \frac{93}{110}
\end{aligned}$$

23. Give the standard form of the equation of the circle with center $(-2, 1)$ and radius 5.

Solution: $(x+2)^2 + (y-1)^2 = 25$

24. Determine the center and radius of a circle having the points $(-1, 7)$ and $(1, 1)$ on the same diameter.

Solution: Since the center is the midpoint of the endpoints of any diameter, plugging the two given points into the midpoint formula will yield the center. Thus, the center is $\left(\frac{-1+1}{2}, \frac{7+1}{2}\right) = \left(\frac{0}{2}, \frac{8}{2}\right) = (0, 4)$.

Plugging in the center and one of the given points (it does not matter which) will yield the radius.

$$\begin{aligned}
r &= \sqrt{(1-0)^2 + (1-4)^2} \\
&= \sqrt{1^2 + (-3)^2} \\
&= \sqrt{1+9} \\
&= \sqrt{10}
\end{aligned}$$

25. A chemist wants to add some water to 60mL of a solution that is 50% sulfuric acid in order to obtain a solution that is 15% sulfuric acid. How much water should she add?

Solution:

	mL liquid	part H_2SO_4	mL H_2SO_4
original	60	0.5	30
water	x	0	0
final	$x + 60$	0.15	$0.15x + 9$

$$\begin{aligned}
0.15x + 9 &= 30 \\
0.15x &= 21 \\
x &= 140\text{mL}
\end{aligned}$$

26. The sum of three consecutive odd integers is 147. What are the integers?

Solution: Let x be the first odd integer, $x+2$ be the second odd integer, and $x+4$ be the third odd integer.

$$\begin{aligned} x + x + 2 + x + 4 &= 147 \\ 3x + 6 &= 147 \\ 3x &= 141 \\ x &= 47 \end{aligned}$$

Thus, the three consecutive odd integers are 47, 49, and 51.

Note that solutions to extra credit questions will **never** appear on a key