MATH 012 Test 2 Key

Note that I cannot graph with this typesetter, so I will try to describe the graphs as best as I can for problems that require graphing.

1. Graph the function $y = 2|x - 3| + 1$ using transformations, starting with the appropriate basic graph. Label each graph with the appropriate equation.

Solution: The basic graph, $y = |x|$, is V-shaped and passes through the origin. The next graph should be $y = |x - 3|$, which is the previous graph shifted 3 to the right. The next graph should be $y = 2|x-3|$, which is the previous graph stretched vertically by a factor of 2. The final graph, that of $y = 2|x - 3| + 1$, is the previous graph shifted 1 up.

2. In the equation $x = y^6$, is y a function of x? Why or why not?

Solution: No, in $x = y^6$, y is not a function of x. In isolating y, the equation $y = \pm\sqrt[6]{x}$ is obtained. (The $\pm$ is necessary because of the even root.)

3. Determine whether each of the following expressions is a polynomial. If an expression is a polynomial, determine its degree.

Solution:

	$1 - (\sqrt{3})x^2 + x^5$	$4x^2 - \|x\|$	$\frac{5}{3}$	$\frac{x^2+1}{x}$
Polynomial	Yes	No	Yes	No
Degree	5	N/A	0	N/A

4. a. Convert the equation $x^2 + y^2 - 6x + 8y = 24$ into standard form.

Solution:

$$\begin{aligned} x^2 + y^2 - 6x + 8y &= 24 \\ x^2 - 6x + y^2 + 8y &= 24 \\ x^2 - 6x + 9 + y^2 + 8y + 16 &= 24 + 9 + 16 \\ (x-3)^2 + (y+4)^2 &= 49 \end{aligned}$$

b. Graph the circle given by your answer to part a.

Solution: The circle given by $(x - 3)^2 + (y + 4)^2 = 49$ has center $(3, -4)$ and radius 7. It passes through the points $(3, 3)$, $(10, -4)$, $(3, -11)$, and $(-4, -4)$.

5. Determine whether the function $h(x) = \dfrac{x^2 + |x|}{x^3}$ is even, odd, or neither.

Solution: $h(x)$ is odd because

$$h(-x) = \frac{(-x)^2 + |-x|}{(-x)^3} = \frac{(-1)^2x^2 + |-1||x|}{(-1)^3x^3} = \frac{x^2 + |x|}{-x^3} = -h(x).$$

6. a. If $f(x) = \dfrac{x+1}{x-2}$ and $g(x) = x^2 - x + 2$, determine $(f \circ g)(x)$.

Solution: $(f \circ g)(x) = f(x^2 - x + 2) = \dfrac{(x^2 - x + 2) + 1}{(x^2 - x + 2) - 2} = \dfrac{x^2 - x + 3}{x^2 - x}$

b. Determine the domain of the function that is your answer to part a.

Solution: Because $\operatorname{dom} g = \mathbb{R}$, it is only required that $x^2 - x \neq 0$. Because $x^2 - x = x(x-1)$, $\operatorname{dom}(f \circ g) = (-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

7. Determine the average rate of change of the function $g(x) = -x^2 + x + 7$ from $x = 1$ to $x = 3$.

Solution:

$$\begin{aligned}
\frac{g(3) - g(1)}{3 - 1} &= \frac{-3^2 + 3 + 7 - (-1^2 + 1 + 7)}{2} \\
&= \frac{-9 + 10 - (-1 + 8)}{2} \\
&= \frac{1 - 7}{2} \\
&= -3
\end{aligned}$$

8. A clothing store decides to sell T-shirts. The cost of materials for each T-shirt is \$5. According to a customer survey, the store will sell 100 of these T-shirts per day if their price is \$15. For each \$1 increase in the price, 5 less T-shirts will sell per day. Let x denote the price of the T-shirts.

a. Determine a function that yields the number of T-shirts sold per day as a function of x.

Solution: We have that $s(x) = -5x + b$ for some $b \in \mathbb{R}$ and that $s(15) = 100$. Since $100 = s(15) = -5(15) + b = -75 + b$, then $b = 175$. Thus, $s(x) = -5x + 175$.

b. Determine a function that yields the daily revenue of selling the T-shirts as a function of x.

Solution: $R(x) = xs(x) = x(-5x + 175) = -5x^2 + 175x$

c. Determine a function that yields the daily cost of selling the T-shirts as a function of x.

Solution: $C(x) = 5s(x) = 5(-5x + 175) = -25x + 875$

d. Determine a function that yields the daily profit of selling the T-shirts as a function of x.

Solution: $P(x) = R(x) - C(x) = -5x^2 + 175x - (-25x + 875) = -5x^2 + 200x - 875$

e. How much should the company charge per T-shirt in order to maximize profit?

Solution: Since $P(x) = -5x^2 + 200x - 875$, then $P(x)$ attains its maximum value when $x = \frac{-200}{2(-5)} = \frac{-200}{-10} = 20$; that is, the price of each shirt is \$20.

f. What is the maximum profit that the company can make per day from selling these T-shirts?

Solution:
$P(20) = -5 \cdot 20^2 + 200 \cdot 20 - 875 = -5 \cdot 400 + 4000 - 875 = -2000 + 3125 = 1125$

Thus, the maximum profit is \$1125.

9. For the graph on the test, determine the domain, range, relative extrema, and intervals of increasing and decreasing.

Solution:

domain: $(-7, 4] \cup [-2, 2]$

range: $[-3, 4)$

relative minima: $(-6, 1), (2, -3)$

relative maxima: $(-2, 3)$

increasing: $(-6, -4)$

decreasing: $(-7, -6) \cup (-2, 2)$

10. Graph the piecewise function $y = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ x^3 - 2 & \text{if } x \geq 0. \end{cases}$

Solution: To the left of the y axis, the graph should look like the basic parabola $y = x^2$ shifted up one unit. There should be an open circle at the point $(0, 1)$. To the right of the y axis, the graph should look like the basic cubic $y = x^3$ shifted down two units. There should be a closed circle at the point $(0, -2)$.

11. Determine the inverse function of $y = \frac{2x - 1}{x + 3}$.

Solution:

$$\begin{aligned} x &= \frac{2y - 1}{y + 3} \\ x(y + 3) &= 2y - 1 \\ xy - 2y &= -3x - 1 \\ y &= \frac{-3x - 1}{x - 2} \end{aligned}$$

12. Determine the end behavior of $p(x) = 1 - 4x^2 + 8x^5$.

Solution: The degree of $p(x)$ is 5, and the leading coefficient is 8. Thus, $p(x)$ is negative towards the left and is positive towards the right.