

### Review Sheet for Test 3

The only new understood instruction on this test is that all asymptotes must be given as lines.

Following is a list of concepts you should know for the test:

- Sections 3.1-3.3: (9 pts.) Use the remainder theorem, the intermediate value theorem, the upper bound theorem, and the lower bound theorem as necessary to obtain information about the graph of a polynomial function.
- Section 3.1: (13 pts.) Graph a polynomial function accurately.
- Section 3.3: (13 pts.) Determine the roots of a polynomial function using synthetic division, factoring, the rational root theorem, Descartes' rule of signs, the upper bound theorem, and the lower bound theorem as necessary.
- Section 3.6: (14 pts.) Graph a rational function accurately.
- Sections 4.1-4.2: (16 pts.) Perform transformations on exponential and/or logarithmic graphs.
- Section 4.1: (7 pts.) Compute interest compounded on a regular basis.  
(6 pts.) Compute interest compounded continuously.
- Section 4.3: (2 pts.) Expand and/or combine logarithms.
- Section 4.4: (8 pts.) Solve exponential and/or logarithmic equations.
- Section 4.5: (12 pts.) Solve word problems involving exponentials and/or logarithms.

## Theorems and Formulas

The intermediate value theorem for polynomials: If  $P(x)$  is a polynomial function,  $a < b$ , and  $P(a) \neq P(b)$ , then, for any real number  $k$  strictly between  $P(a)$  and  $P(b)$ , there exists a real number  $c$  with  $a < c < b$  such that  $P(c) = k$ . (Note that  $k = 0$  is used most frequently in this class.)

The remainder theorem: If  $P(x)$  is a polynomial and  $c$  is a real number, then  $P(c)$  is equal to the remainder from dividing  $P(x)$  by  $x - c$ .

The rational root theorem: If  $P(x)$  is a polynomial function with integer coefficients, the only possibilities for rational roots of  $P(x)$  are of the form  $\frac{p}{q}$ , where  $p$  is a factor of the constant term of  $P(x)$  and  $q$  is a factor of the leading coefficient of  $P(x)$ .

Descartes' rule of signs: If  $P(x)$  is a polynomial function, then the number of positive real roots of  $P(x)$  is equal to the number of variations in sign of  $P(x)$  or is less than that by some multiple of 2. (For the number of negative real roots of  $P(x)$ , a similar procedure is used on  $P(-x)$ .)

The upper bound theorem: If  $P(x)$  is a polynomial function and  $c > 0$  such that, when  $P(x)$  is synthetically divided by  $x - c$ , the bottom row consists solely of nonnegative numbers, then none of the real roots of  $P(x)$  is larger than  $c$ .

The lower bound theorem: If  $P(x)$  is a polynomial function and  $c < 0$  such that, when  $P(x)$  is synthetically divided by  $x - c$ , the bottom row consists of numbers alternating in sign, then none of the real roots of  $P(x)$  is smaller than  $c$ .

Interest compounded on a regular basis: Let  $P$  be the principal,  $r$  be the interest rate (as a decimal),  $n$  be the number of compoundings per year,  $t$  be the time elapsed (in years), and  $A$  be the final amount. Then

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}.$$

Interest compounded continuously: Let  $P$  be the principal,  $r$  be the interest rate (as a decimal),  $t$  be the time elapsed (in years), and  $A$  be the final amount. Then

$$A = Pe^{rt}.$$

Algebraic properties of logarithms:  $b^x = a$  iff  $\log_b a = x$

$$b^{\log_b x} = x$$

$$\log_b(mn) = \log_b m + \log_b n$$

$$\log_b \left( \frac{m}{n} \right) = \log_b m - \log_b n$$

$$\log_b(m^n) = n \log_b m$$

Change of base for logarithms:  $\log_c x = \frac{\log_b x}{\log_b c}$