

Transformations

In general, when a number is grouped with the x (such as in $(x - 3)^3$ or $\sqrt{2x}$), it affects the graph in a horizontal manner, and when a number is not grouped with the x (such as in $x^3 - 3$ or $2\sqrt{x}$), it affects the graph in a vertical manner.

Addition and subtraction of a constant always shift the graph in some direction.

Multiplication by a negative always reflects the graph about an axis.

Multiplication by a number other than 1 and -1 always stretches or compresses the graph in some direction.

Note that the vertical shifts, stretches, and compressions work intuitively, whereas the horizontal shifts, stretches, and compressions do not.

Below is a summary of the types of transformations and how they are achieved.

Type	Horizontal Version	Vertical Version
Shift	The graph of $f(x - b)$ is that of $f(x)$ shifted b units to the right	The graph of $f(x) + b$ that of $f(x)$ that of $f(x)$ shifted b units up
Stretch	The graph of $f(cx)$ is that of $f(x)$ stretched by a factor of $\frac{1}{ c }$ when $0 < c < 1$	The graph of $cf(x)$ is that of $f(x)$ stretched by a factor of $ c $ when $ c > 1$
Shrink	The graph of $f(cx)$ is that of $f(x)$ compressed by a factor of $ c $ when $ c > 1$	The graph of $cf(x)$ is that of $f(x)$ compressed by a factor of $\frac{1}{ c }$ when $0 < c < 1$
Reflect	The graph of $f(-x)$ is that of $f(x)$ reflected about the y axis	The graph of $-f(x)$ is that of $f(x)$ reflected about the x axis