Math 347 C1

HOUR EXAM III

2 August 2006

<u>SOLUTIONS</u>

1. Solve: $a_n = 3a_{n-1} + 1, \quad a_0 = 2.$

   <u>SOLUTION</u>: According to Theorem 12.16, $a_n = A3^n + p(n)$, where $p(n)$ is a polynomial in n of degree 0.
   Therefore, let $a_n = A3^n + B$ and apply the recurrence. Thus $A3^n + B = 3(A3^{n-1} + B) + 1$ and we have $A3^n + B = A3^n + 3B + 1$. Hence $B = -\frac{1}{2}$, giving

   $$a_n = A3^n - \frac{1}{2}.$$

   We use the initial condition $a_0 = 2$ to find A, that is, $a_0 = 2 = A3^0 - \frac{1}{2}$ and so $A = \frac{5}{2}$. The final answer is

   $$a_n = \frac{5}{2}3^n - \frac{1}{2}.$$

2. Let $f : I \to R$, with I an open interval containing a, and R, the real number system. State the definition of

   $$lim_{x\to a} f(x) = L.$$

   What does it mean to say the the function f above is *continuous* at the point a?

   <u>SOLUTION</u>: $lim_{x\to a} f(x) = L$ means

   $$(\forall \epsilon > 0)(\exists \delta > 0)(0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon.)$$

   f is *continuous* at the point a means that $lim_{x\to a} f(x) = L$, that is, the limit exists, and that $L = f(a)$.

3 . Express $0.5151\ldots = \overline{.51}$ as a rational number in lowest terms.

   <u>SOLUTION</u>: $100(\overline{.51}) = 51.5151\ldots = 51 + \overline{.51}$. Therefore $99(\overline{.51}) = 51$. Thus

   $$\overline{.51} = \frac{51}{99} = \frac{17}{33}.$$

4. Let S be a nonempty set of real numbers. Show that if L is a lower bound for S and $L \in S$, then L is the greatest lower bound for S.

   SOLUTION: Suppose that L is not the greatest lower bound. Then there is another lower bound for S, say M, with $M > L$. But $L \in S$ and since $M > L$, M is not a lower bound for S. This is a contradiction and so our assumption that L is not the greatest lower bound for S is false. Thus since L is a lower bound, it must be the greatest lower bound.