

Douglas B. West - Abstracts since 1992

188. Extremal graphs with a given number of perfect matchings (with S.G. Harke, D. Stolee, and M. Yancey; submitted - 23 pages).

Let $f(n, p)$ denote the maximum number of edges in a graph having n vertices and exactly p perfect matchings. For fixed p , Dudek and Schmitt showed that $f(n, p) = n^2/4 + c_p$ for some constant c_p when n is at least some constant n_p . For $p \leq 6$, they also determined c_p and n_p . For fixed p , we show that the extremal graphs for all n are determined by those with $O(\sqrt{p})$ vertices. As a corollary (after a computer search), we determine c_p and n_p for $p \leq 10$. We also present lower bounds on $f(n, p)$ proving that $c_p > 0$ for $p \geq 2$ (as conjectured by Dudek and Schmitt), and we conjecture an upper bound on $f(n, p)$. Our structural results are based on Lovász's Cathedral Theorem.

187. Degree Ramsey numbers of graphs (with W.B. Kinnersley and K.G. Milans; submitted - 26 pages). The s -color degree Ramsey number of a graph G , written $R_\Delta(G; s)$, is the least value of $\Delta(H)$ among the graphs H such that every s -edge-coloring of H contains a monochromatic copy of G . If T is a tree in which one vertex has degree at most k and all others have degree at most $\lceil k/2 \rceil$, then $R_\Delta(T; s) = s(k-1) + \varepsilon$, where $\varepsilon = 1$ when k is odd and $\varepsilon = 0$ when k is even. For general trees, $R_\Delta(T; s) \leq 2s(\Delta(T) - 1)$. For sharpness of the upper bound, consider the *double-star* $S_{a,b}$, the tree whose two non-leaf vertices have degrees a and b . If $a \leq b$, then $R_\Delta(S_{a,b}; 2)$ is $2b - 2$ when $a < b$ and b is even; it is $2b - 1$ otherwise. If s is fixed and at least 3, then $R_\Delta(S_{b,b}; s) = f(s)(b-1) - o(b)$, where $f(s) = 2s - 3.5 - O(s^{-1})$. We also prove several results about edge-colorings of bounded-degree graphs that are related to degree Ramsey numbers of paths. Finally, for cycles we show that $R_\Delta(C_{2k+1}; s) \geq 2^s + 1$, that $R_\Delta(C_{2k}; s) \geq 2s$, and that $R_\Delta(C_4; 2) = 5$.

186. Permutation bigraphs: An analogue of permutation graphs (with P.K. Saha, A. Basu, and M.K. Sen; submitted - 15 pages).

We introduce the class of permutation bigraphs as an analogue of permutation graphs. We show that this is precisely the class of bigraphs having Ferrers dimension at most 2. We also characterize the subclasses of interval bigraphs and indifference bigraphs in terms of their permutation labelings, and we relate permutation bigraphs to posets of dimension 2.

185. Vertex degrees in outerplanar graphs (with K.F. Jao; submitted - 10 pages).

For an outerplanar graph on n vertices, we determine the maximum number of vertices of degree at least k . For $k = 4$ (and $n \geq 7$) the answer is $n - 4$. For $k = 5$ (and $n \geq 4$), the answer is $\lfloor \frac{2(n-4)}{3} \rfloor$ (except one less when $n \equiv 1 \pmod{6}$). For $k \geq 6$ (and $n \geq k + 2$), the answer is $\lfloor \frac{n-6}{k-4} \rfloor$. We also determine the maximum sum of the degrees of s vertices in an n -vertex outerplanar graph and the maximum sum of the degrees of the vertices with degree at least k .

184. Equitable hypergraph orientations (with Y. Caro and R. Juster; submitted - 5 pages).

An *orientation* of a hypergraph chooses for each edge a linear ordering of its vertices. For $1 \leq p < r$, an orientation of an r -uniform hypergraph is p -equitable if each p -set of vertices occupies each p -set of positions in about the same number of edges. We prove that every r -uniform hypergraph has 1-equitable and $(r-1)$ -equitable orientations. The special case $r = 2$ (graphs) is well known, saying that some orientation has indegree and outdegree differing by at most 1 at each vertex. For $1 < p < r - 1$, we prove a necessary condition, implying that some complete r -uniform hypergraphs have no p -equitable orientation. We conjecture that when p is fixed and each p -set of vertices appears in at most k edges, p -equitable orientations always exist when r is sufficiently large. We use the Local Lemma to prove that large enough r ensures an orientation that is "nearly" p -equitable, with each p -set of vertices occupying each p -set of positions at most twice.

183. Chain-making games in grid-like posets (with D.W. Cranston, W.B. Kinnersley, K.G. Milans, and G.J. Puleo; submitted - 14 pages).

We study the Maker-Breaker game on the hypergraph of chains of fixed size in a poset. In a product of chains, the maximum size of a chain that Maker can guarantee building is $k - \lfloor r/2 \rfloor$, where k is the maximum size of a

chain in the product, and r is the maximum size of a factor chain. We also study a variant in which Maker must follow the chain in order, called the *Walker-Blocker game*. In the poset consisting of the bottom k levels of the product of d arbitrarily long chains, Walker can guarantee a chain that hits all levels if $d \geq 14$; this result uses a solution to Conway's Angel-Devil game. When $d = 2$, the maximum that Walker can guarantee is only $2/3$ of the levels, and $2/3$ is asymptotically achievable in the product of two equal chains.

182. Length thresholds for graphic lists given fixed largest and smallest entries and bounded gaps (with M.D. Barrus, S.G. Hartke, and K.F. Jao; submitted - 14 pages).

In a list of positive integers, let r and s denote the largest and smallest entries. A list is *gap-free* if each integer between r and s is present. We prove that a gap-free even-summed list is graphic if it has at least $r + \frac{r+s+1}{2s}$ terms.

With no restriction on gaps, length at least $\frac{(r+s+1)^2}{4s}$ suffices, as proved by Zverovich and Zverovich. Both bounds are sharp within 1. When the gaps between consecutive terms are bounded by g , we prove a more general length threshold that includes both of these results. As a tool, we prove that if an even-summed positive list d has no repeated entries other than r and s (and the length exceeds r), then to prove that d is graphic it suffices to check only the ℓ th Erdős–Gallai inequality, where $\ell = \max\{k: d_k \geq k\}$.

181. Decomposition of sparse graphs into forests and a graph with bounded degree (with S.-J. Kim, A.V. Kostochka, H. Wu, and X. Zhu; submitted - 24 pages).

Say that a graph with maximum degree at most d is *d-bounded*. For $d > k$, we prove a sharp sparseness condition for decomposition into k forests and a d -bounded graph. The condition holds for every graph with fractional arboricity at most $k + \frac{d}{k+d+1}$. For $k = 1$, it also implies that every graph with maximum average degree less than $2 + \frac{2d}{d+2}$ decomposes into one forest and a d -bounded graph. When $d = k + 1$, and when $k = 1$ and $d \leq 6$, the d -bounded graph in the decomposition can also be required to be a forest, and for $(k, d) = (1, 2)$ it can also be required to have at most d edges in each component. For $d \leq k$, we prove that every graph with fractional arboricity at most $k + \frac{d}{2k+2}$ decomposes into $k + 1$ forests, of which one is d -bounded.

180. Acyclic sets in k -majority tournaments (with K.G. Milans and D.H. Schreiber; submitted - 8 pages). When Π is a set of k linear orders on a ground set X , and k is odd, the *k-majority tournament* generated by Π has vertex set X and has an edge from u to v if and only if a majority of the orders in Π rank u before v . Let $f_k(n)$ be the minimum, over all k -majority tournaments with n vertices, of the maximum order of an induced transitive subtournament. We prove that $f_3(n) \geq \sqrt{n}$ always and that $f_3(n) \leq 2\sqrt{n} - 1$ when n is a perfect square. We also prove that $f_5(n) \geq n^{1/4}$. For general k , we prove that $n^{c_k} \leq f_k(n) \leq n^{d_k}$, where $c_k = 3^{-(k-1)/2}$ and $d_k = \frac{(\ln \ln k) + 2}{(\ln k) - 1} (1 + \frac{\ln k}{n})$.

179. The A_4 -structure of a graph (with M.D. Barrus; submitted - 18 pages).

We define the A_4 -structure of a graph G to be the 4-uniform hypergraph on the vertex set of G whose edges are the vertex subsets inducing $2K_2$, C_4 , or P_4 . We show how the A_4 -structure is naturally related to the canonical decomposition of a graph as defined by Tyshkevich [Discrete Mathematics 220 (2000) 201-238]; in particular, a graph is indecomposable if and only if its A_4 -structure is connected. We show that the A_4 -structure satisfies several analogues of results on the P_4 -structure of a graph. Motivated by the problem of generating all graphs having a given A_4 -structure, we characterize those graphs having the same A_4 -structure as a split graph.

178. Bounds on the k -dimension of products of special posets (with M. Baym; submitted - 11 pages).

Trotter conjectured that $\dim P \times Q \geq \dim P + \dim Q - 2$ for all posets P and Q . To shed light on this, we study the k -dimension of products of finite orders, where the k -dimension $\dim_k(P)$ is the minimum t such that P embeds in the product of t k -element chains. For fixed k , the value $2\dim_k(P) - \dim_k(P \times P)$ is unbounded when P is an antichain, and $2\dim 2(mP) - \dim 2(mP \times mP)$ is unbounded when P is a fixed poset with unique maximum and minimum. For products of the "standard" orders S_m and S_n of dimensions m and n , $\dim_k(S_m \times S_n) = m + n - \min\{2, k - 2\}$. For higher order products of "standard" orders, $\dim 2(\prod_{i=1}^t S_{n_i}) = \sum n_i$ if each $n_i \geq t$.

177. The induced Turán problem; large P_m -free graphs with bounded degree (with M.S. Chung and T. Jiang; submitted - 22 pages).

A graph is *H-free* if it has no induced subgraph isomorphic to H . Let $\text{ex}^*(D; H)$ be the maximum number of edges in an H -free connected graph with maximum degree D ; this is finite if and only if H is a disjoint union of paths. Earlier results include $\text{ex}^*(D; P_4) = D^2$ and the exact computation of $\text{ex}^*(D; 2P_3)$. For fixed $m \geq 6$, we prove that $\text{ex}^*(D; P_m) = \frac{1}{2}D^{m/2} + O(D^{m/2-1})$ when m is even and $\text{ex}^*(D; P_m) = \frac{1}{8}D^{(m+1)/2} + O(D^{(m-1)/2})$ when m is odd (we also find the exact maximum number of vertices in such a graph when m is odd). Finally, we obtain $\text{ex}^*(D; P_5)$ exactly when $D \geq 187$: for example, it equals $\frac{2}{27}D^3 + \frac{7}{18}D^2 + \frac{1}{2}D$ when D is divisible by 3.

176. On-line Ramsey Theory for Bounded Degree Graphs (with J. Butterfield, T. Grauman, W.B. Kinnersley, K.G. Milans and C.J. Stocker; submitted - 22 pages).

When graph Ramsey theory is viewed as a game, “Painter” 2-colors the edges of a graph presented by “Builder”. Builder wins if every coloring has a monochromatic copy of a fixed graph G . In the on-line version, Builder presents edges one by one, and Painter must color each edge without seeing later edges. Builder must keep the presented graph in a class \mathcal{H} . Builder wins the game (G, \mathcal{H}) if a monochromatic copy of G can be forced. The *on-line degree Ramsey number* $\mathring{R}_\Delta(G)$ is the least k such that Builder wins (G, \mathcal{H}) when \mathcal{H} is the class of graphs with maximum degree at most k . Among our results are the following.

- 1) $\mathring{R}_\Delta(G) \leq 3$ if and only if G is a linear forest or each component is contained in $K_{1,3}$.
- 2) $\mathring{R}_\Delta(G) \geq \Delta(G) + t - 1$, where $t = \max_{uv \in E(G)} \min\{d(u), d(v)\}$.
- 3) $\mathring{R}_\Delta(G) \leq d_1 + d_2 - 1$ for a tree G , where d_1, \dots, d_n is the nonincreasing degree list.
- 4) $4 \leq \mathring{R}_\Delta(C_n) \leq 5$.
- 5) $\mathring{R}_\Delta(C_n) = 4$ when n is even or 3 or at least 689.
- 6) $\mathring{R}_\Delta(G) \leq 6$ when $\Delta(G) \leq 2$.

The lower bounds come from strategies for Painter that color every edge red as long as it keeps the red graph in a specified class. The upper bounds use a result allowing Builder to assume that Painter plays “consistently”.

175. Spanning cycles through specified edges in bipartite graphs (with R. Zamani; submitted - 16 pages).

Pósa proved that if G is an n -vertex graph in which any two nonadjacent vertices have degree sum at least $n + k$, then G has a spanning cycle containing any specified family of disjoint paths with a total of k edges. We consider the analogous problem for a bipartite graph G with n vertices and parts of equal size. Let F be a subgraph of G whose components are nontrivial paths. Let k be the number of edges in F , and let t_1 and t_2 be the numbers of components of F having odd and even length, respectively. We prove that G has a spanning cycle containing F if any two nonadjacent vertices in opposite partite sets have degree-sum at least $n/2 + \tau(F)$, where $\tau(F) = \lceil k/2 \rceil + \epsilon$ (here $\epsilon = 1$ if $t_1 = 0$ or if $(t_1, t_2) \in \{(1, 0), (2, 0)\}$, and $\epsilon = 0$ otherwise). We show also that this threshold on the degree-sum is sharp when $n > 3k$.

174. Cycle spectra of Hamiltonian graphs (with K. G. Milans, D. Rautenbach, and F. Regen; submitted - 16 pages).

We prove that every graph consisting of a spanning cycle plus q chords has cycles of at least $\sqrt{(2/3 - o(1))q}$ different lengths. The coefficient $2/3$ cannot be increased above 1, since when $q = n^2/4 - n$ there are exactly $\sqrt{q+1}$ lengths of cycles in $K_{n/2, n/2}$. For general m and n , there are Hamiltonian graphs with n vertices and m edges having at most $2\lceil \sqrt{m - n + 1} \rceil$ different cycle lengths.

173. Packing of Steiner trees and S -connectors in graphs (with H. Wu; submitted - 28 pages).

Nash-Williams and Tutte independently characterized when a graph has k edge-disjoint spanning trees; a consequence is that $2k$ -edge-connected graphs have k edge-disjoint spanning trees. Kriesell conjectured a more general statement: defining a set $S \subseteq V(G)$ to be *j -edge-connected in G* if S lies in a single component of any graph obtained by deleting fewer than j edges from G , he conjectured that if S is $2k$ -edge-connected in G , then G has k edge-disjoint trees containing S . Lap-Chi Lau proved that the conclusion holds whenever S is $24k$ -edge-connected in G . We improve Lau’s result by showing that it suffices for S to be $6.5k$ -edge-connected in G . This and an analogous result for packing stronger objects called “ S -connectors” follow from a common generalization of the

Tree Packing Theorem and Hakimi's criterion for orientations with specified outdegrees. We prove the general theorem using submodular functions and the Matroid Union Theorem.

172. Longest cycles in k -connected graphs with given independence number (with S. O and H. Wu; *J. Comb. Theory (B)* - in press, 8 pages).

The Chvátal–Erdős Theorem states that every graph whose connectivity is at least its independence number has a spanning cycle. In 1976, Fouquet and Jolivet conjectured an extension: If G is an n -vertex k -connected graph with independence number a , and $a \geq k$, then G has a cycle with length at least $\frac{k(n+a-k)}{a}$. We prove this conjecture.

171. Uniquely C_4 -Saturated Graphs (with J. Cooper, J. Lenz, T.D. LeSaulnier, and P.S. Wenger; *Graphs and Combin.* - in press, 9 pages).

For a fixed graph H , a graph G is *uniquely H -saturated* if G does not contain H , but the addition of any edge from \overline{G} to G completes exactly one copy of H . Using a combination of algebraic methods and counting arguments, we determine all the uniquely C_4 -saturated graphs; there are only ten of them.

170. Packing of graphic sequences (with A. Busch, M. Ferrara, S. G. Hartke, M. S. Jacobson, and H. Kaul; *J. Graph Theory* - in press, 11 pages).

Let π_1 and π_2 be graphic n -tuples, with $\pi_1 = (d_1^{(1)}, \dots, d_n^{(1)})$ and $\pi_2 = (d_1^{(2)}, \dots, d_n^{(2)})$ (they need not be monotone). We say that π_1 and π_2 *pack* if there exist edge-disjoint graphs G_1 and G_2 with vertex set $\{v_1, \dots, v_n\}$ such that the degrees of v_i in G_1 and G_2 are $d_i^{(1)}$ and $d_i^{(2)}$, respectively. We prove that two graphic n -tuples pack if $\Delta \leq \sqrt{2\delta n} - (\delta - 1)$, where Δ and δ denote the largest and smallest entries in $\pi_1 + \pi_2$ (strict inequality when $\delta = 1$); furthermore, the bound is sharp.

Kundu proved that a graphic n -tuple π is realizable by a graph having a k -factor if the list obtained by subtracting k from each entry is graphic; Brualdi conjectured that when n is even some realization contains k edge-disjoint 1-factors. We prove the conjecture when the largest entry is at most $n/2 + 1$ and also when $k \leq 3$.

169. A short proof of the Berge–Tutte Formula and the Gallai–Edmonds Structure Theorem (*European J. Comb.* - in press, 5 pages).

We present a short proof of the Berge–Tutte Formula and the Gallai–Edmonds Structure Theorem from Hall's Theorem.

168. Inequalities of Nordhaus–Gaddum type for connected domination (with H. Karami, A. Khodkar, and S.M. Sheikholeslami; *Graphs and Combin.* - in press, 8 pages).

A set S of vertices of a graph G is a *connected dominating set* if every vertex not in S is adjacent to some vertex in S and the subgraph induced by S is connected. The *connected domination number* $\gamma_c(G)$ is the minimum size of a connected dominating set of G . In this paper we prove that $\gamma_c(G) + \gamma_c(\overline{G}) \leq \min\{\delta(G), \delta(\overline{G})\} + 4$ for every n -vertex graph G such that G and \overline{G} have diameter 2. The bound is sharp for each value of the right side. Also, $\gamma_c(G) + \gamma_c(\overline{G}) \leq 3n/4$ if G and \overline{G} are connected, have minimum degree at least 3, and $n \geq 14$. Finally, we prove that $\gamma_c(G) + \gamma_c(\overline{G}) \leq \min\{\delta(G), \delta(\overline{G})\} + 2$ if $\gamma_c(G), \gamma_c(\overline{G}) \geq 4$ and show that the bound is sharp when $\min\{\delta(G), \delta(\overline{G})\} = 6$.

167. Overlap Number of Graphs (with D.W. Cranston, N. Korula, T.D. LeSaulnier, K.G. Milans, C. Stocker, and J. Vandenbussche; *J. Graph Theory* - in press, 21 pages).

An *overlap representation* of a graph G assigns sets to vertices so that vertices are adjacent if and only if their assigned sets intersect with neither containing the other. The *overlap number* $\phi(G)$ (introduced by Rosgen) is the minimum size of the union of the sets in such a representation. We prove the following: (1) An optimal overlap representation of a tree can be produced in linear time, and its size is the number of vertices in the largest subtree in which the neighbor of any leaf has degree 2. (2) If $\delta(G) \geq 2$ and $G \neq K_3$, then $\phi(G) \leq |E(G)| - 1$, with equality when G is connected and triangle-free and has no star-cutset. (3) If G is an n -vertex plane graph with $n \geq 5$, then $\phi(G) \leq 2n - 5$, with equality when every face has length 4 and there is no star-cutset. (4) If G is an n -vertex graph with $n \geq 14$, then $\phi(G) \leq n^2/4 - n/2 - 1$, with equality for even n when G arises from $K_{n/2, n/2}$ by deleting a perfect matching.

166. A new proof of 3-colorability of Eulerian triangulations (with M.-T. Tsai; *Ars Mathematicae Contemp.* - in press, 6 pages).

Using the existence of noncrossing Eulerian circuits in Eulerian plane graphs, we give a short constructive proof of the theorem of Heawood that Eulerian triangulations are 3-colorable.

165. Matching and edge-connectivity in regular graphs (with S. O; *European J. Comb.* 32 (2011), 324–329).

Henning and Yeo proved a lower bound for the minimum size of a maximum matching in a connected k -regular graphs with n vertices; it is sharp infinitely often. In an earlier paper, we characterized when equality holds. In this paper, we prove a lower bound for the minimum size of a maximum matching in an l -edge-connected k -regular graph with n vertices, for $l \geq 2$ and $k \geq 4$. Again it is sharp for infinitely many n , and we characterize when equality holds in the bound.

164. Degree-splittability of multigraphs and caterpillars (with J.O. Choi and L. Ozkahya; *Proc. 42th S.E. Intl. Conf. Graph Th. Comb. Comp. Congr. Numer.* 202 (2010), 137–147).

A multigraph is *degree-splittable* if it decomposes into two submultigraphs whose degree lists are the same, plus one leftover edge when the total number of edges is odd. We prove that a multigraph is splittable if the number of vertices of each odd degree is even. We also study degree-splittability of caterpillars, characterizing the splittable caterpillars with diameter at most 4, providing a sufficient condition for splittability of caterpillars in general, and proving that the smallest maximum degree of a non-splittable caterpillar is 5.

163. The edge-count criterion for graphic lists (with G. Isaak; *Electronic J. Comb.* 17 (2010), Paper #N36, 4 pages). We give a new short proof of Koren's characterization of graphic lists, extended to multigraphs with bounded multiplicity p , called *p-graphs*. The Edge-Count Criterion (ECC) for an integer n -tuple d and integer p is the statement that for all disjoint sets I and J of indices, $\sum_{i \in I} d_i + \sum_{j \in J} [p(n-1) - d_j] \geq p|I||J|$. An integer list d is the degree list of a p -graph if and only if it has even sum and satisfies ECC. Analogous statements hold for bipartite or directed graphs.

162. Degree-associated reconstruction number of graphs (with M. D. Barrus; *Discrete Math.* 310 (2010), 2600–2612).

A *card* of a graph G is a subgraph formed by deleting one vertex. The Reconstruction Conjecture states that each graph with at least three vertices is determined by its multiset of cards. A *dacard* specifies the degree of the deleted vertex along with the card. The *degree-associated reconstruction number* $\text{drn}(G)$ is the minimum number of dacards that determine G . We show that $\text{drn}(G) = 2$ for almost all graphs, determine the graphs with $\text{drn}(G) = 1$, show that $\text{drn}(G) \geq 3$ when G is vertex-transitive and give a sufficient condition for equality, construct vertex-transitive graphs with large drn , and prove that $\text{drn}(G) = 2$ for all caterpillars except stars and one 6-vertex example. We conjecture that $\text{drn}(G) \leq 2$ for all but finitely many trees.

161. Forbidden subposets for fractional weak discrepancy at most k (with J. O. Choi; *European J. Comb.* 31 (2010), 1957–1963).

The *fractional weak discrepancy* of a poset P , written $\text{wd}_F(P)$, is the least k such that some $f: P \rightarrow \mathbb{R}$ satisfies $f(y) - f(x) \geq 1$ for $x \prec y$ and $|f(y) - f(x)| \leq k$ for $x \parallel y$. We determine the minimal forbidden subposets for the property $\text{wd}_F(P) \leq k$ when k is an integer.

160. Rainbow matching in edge-colored graphs (with T.D. LeSaulnier, C. Stocker, and P.S. Wenger; *Electronic J. Comb.* 17 (2010), Paper N26, 5 pages).

A *rainbow subgraph* of an edge-colored graph is a subgraph whose edges have distinct colors. The *color degree* of a vertex v is the number of different colors on edges incident to v . Wang and Li conjectured that for $k \geq 4$, every edge-colored graph with minimum color degree at least k contains a rainbow matching of size at least $\lceil k/2 \rceil$. We prove the slightly weaker statement that a rainbow matching of size at least $\lfloor k/2 \rfloor$ is guaranteed. We also give sufficient conditions for a rainbow matching of size at least $\lceil k/2 \rceil$ that fail to hold only for finitely many exceptions (for each odd k).

159. Decomposition of sparse graphs, with application to game coloring number (with M. Montassier, A. Pêcher, A. Raspaud, and X. Zhu; *Discrete Math.* 310 (2010), 1520–1523).

Let k be a nonnegative integer, and let $m_k = \frac{4(k+1)(k+3)}{k^2+6k+6}$. We prove that every simple graph with maximum average degree less than m_k decomposes into a forest and a subgraph with maximum degree at most k (furthermore, when $k \leq 3$ both subgraphs can be required to be forests). It follows that every simple graph with maximum average degree less than m_k has game coloring number at most $4 + k$.

158. A short constructive proof of the Erdős–Gallai characterization of graphic lists (with A. Tripathi and S. Venugopalan; *Discrete Math.* 310 (2010), 843–844).

Erdős and Gallai proved that a nonincreasing list (d_1, \dots, d_n) of nonnegative integers is the list of degrees of a graph (with no loops or multi-edges) if and only if the sum is even and the list satisfies $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min\{k, d_i\}$ for $1 \leq k \leq n$. We give a short constructive proof of the characterization.

157. Balloons, cut-edges, matchings, and total domination in regular graphs of odd degree (with Suil O; *J. Graph Theory* 64 (2010), 116–131).

A *balloon* in a graph G is a maximal 2-edge-connected subgraph incident to exactly one cut-edge of G . Let $b(G)$ be the number of balloons, let $c(G)$ be the number of cut-edges, and let $\alpha'(G)$ be the maximum size of a matching. Let \mathcal{F}_n be the family of connected $(2r+1)$ -regular graphs with n vertices, and let $b = \max\{b(G) : G \in \mathcal{F}_n\}$. For $G \in \mathcal{F}_n$, we prove the sharp inequalities $c(G) \leq \frac{r(n-2)-2}{2r^2+2r-1} - 1$ and $\alpha'(G) \geq \frac{n}{2} - \frac{rb}{2r+1}$. Using $b \leq \frac{(2r-1)n+2}{4r^2+4r-2}$, we obtain a simple proof of the bound $\alpha'(G) \geq \frac{n}{2} - \frac{r}{2} \frac{(2r-1)n+2}{(2r+1)(2r^2+r-1)}$ proved by Henning and Yeo. For each of these bounds and each r , the approach using balloons allows us to determine the infinite family where equality holds. For the total domination number $\gamma_t(G)$ of a cubic graph, we prove $\gamma_t(G) \leq \frac{n}{2} - \frac{b(G)}{2}$ (except that $\gamma_t(G)$ may be $n/2 - 1$ when $b(G) = 3$ and the balloons cover all but one vertex). With $\alpha'(G) \geq \frac{n}{2} - \frac{b(G)}{3}$ for cubic graphs, this improves the known inequality $\gamma_t(G) \leq \alpha'(G)$.

156. Oriented diameter of graphs with diameter 3 (with P.K. Kwok and Q. Liu; *J. Comb. Theory (B)* 100 (2010), 265–274).

In 1978, Chvátal and Thomassen proved that every 2-edge-connected graph with diameter 2 has an orientation with diameter at most 6. They also gave general bounds on the smallest value $f(d)$ such that every 2-edge-connected graph G with diameter d has an orientation with diameter at most $f(d)$. For $d = 3$, their general bounds reduce to $8 \leq f(3) \leq 24$. We improve these bounds to $9 \leq f(3) \leq 12$.

155. Extremal problems for Roman domination (with E.W. Chambers, B. Kinnersley, and N. Prince; *SIAM J. Discr. Math.* 23 (2009), 1575–1586). A *Roman dominating function* of a graph G is a labeling $f: V(G) \rightarrow \{0, 1, 2\}$ such that every vertex with label 0 has a neighbor with label 2. The *Roman domination number* $\gamma_R(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over such functions. Let G be a connected n -vertex graph. We prove that $\gamma_R(G) \leq 4n/5$, and we characterize the graphs achieving equality. We obtain sharp upper and lower bounds for $\gamma_R(G) + \gamma_R(\overline{G})$ and $\gamma_R(G)\gamma_R(\overline{G})$, improving known results for domination number. We prove that $\gamma_R(G) \leq 8n/11$ when $\delta(G) \geq 2$ and $n \geq 9$, and this is sharp. We conjecture that $\gamma_R(G) \leq \lceil 2n/3 \rceil$ when G is 2-connected, which we prove for a special class.

154. Matching extendibility in hypercubes (with J. Vandenbussche; *SIAM J. Discr. Math.* 23 (2009), 1539–1547).

In a bipartite graph G , a set $S \subseteq V(G)$ is *deficient* if $|N(S)| < |S|$. A matching M (with vertex set U) is *k -suitable* if $G - U$ has no deficient set of size less than k . Let $f_k(d)$ be the maximum r such that in the d -dimensional hypercube Q_d every k -suitable matching having size at most r extends to a perfect matching. We generalize results of Limaye and Sarvate by proving that $f_k(d) = k(d-k) + \binom{k-1}{2}$ for $k \leq d-3$. To this end we prove lower bounds on the sizes of neighborhoods of vertex sets in Q_d . We also prove that every induced matching in Q_d extends to a perfect matching.

153. Pagenumber of k -trees (with J.R. Vandebussche and G. Yu; *SIAM J. Discr. Math.* 23 (2009), 1455–1464).

A *book embedding* of a graph can be viewed as ordering the vertices around a circle and then embedding the edges as chords on layers, with crossing chords on different layers. The *pagenumber* is the minimum number of layers in such an embedding. A k -tree is a graph that can be obtained from the complete graph K_k by iteratively adding one vertex whose neighborhood in the current graph is a clique. Ganley and Heath proved that k -trees have pagenumber at most $k + 1$, and they conjectured that the best possible bound is k . We disprove this conjecture, showing for all k that there exist k -trees with pagenumber $k + 1$. We also provide an algorithm that produces an embedding in k pages for a special class of k -trees.

152. Classes of 3-regular graphs that are (7,2)-edge-choosable (with D.W. Cranston; *SIAM J. Discr. Math.* 23 (2009), 872–881). A graph is (7,2)-edge-choosable if, for every assignment of lists of size 7 to the edges, it is possible to choose two colors for each edge from its list so that no color is chosen for two incident edges. We show that every 3-edge-colorable graph is (7,2)-edge-choosable and also that many non-3-edge-colorable 3-regular graphs are (7,2)-edge-choosable.

151. Chromatic number for a generalization of Cartesian product graphs (with D. Král'; *Electronic J. Comb.* 16 (2009), Paper #R71, 9 pages).

Let \mathcal{G} be a class of graphs. The d -fold grid over \mathcal{G} , denoted \mathcal{G}^d , is the family of graphs obtained from d -dimensional rectangular grids of vertices by placing a graph from \mathcal{G} on each of the lines parallel to one of the axes. Thus each vertex belongs to d of these subgraphs. Let $f(\mathcal{G}; d) = \max_{G \in \mathcal{G}^d} \chi(G)$. If each graph in \mathcal{G} is k -colorable, then $f(\mathcal{G}; d) \leq k^d$. We show that this bound is best possible by proving that $f(\mathcal{G}; d) = k^d$ when \mathcal{G} is the class of all k -colorable graphs. We also show that $f(\mathcal{G}; d) \geq \left\lfloor \sqrt{\frac{d}{6 \log d}} \right\rfloor$ when \mathcal{G} is the class of graphs with at most one edge, and $f(\mathcal{G}; d) \geq \left\lfloor \frac{d}{6 \log d} \right\rfloor$ when \mathcal{G} is the class of graphs with maximum degree 1.

150. Proper path-factors and interval edge-coloring of (3,4)-biregular bigraphs (with A.S. Asratian, C.J. Casselgren, and J. Vandebussche; *J. Graph Theory* 61 (2009), 88–97).

An *interval coloring* of a graph G is a proper coloring of $E(G)$ by positive integers such that the colors on the edges incident to any vertex are consecutive. A (3,4)-biregular bigraph is a bipartite graph in which each vertex of one part has degree 3 and each vertex of the other has degree 4; it is unknown whether these all have interval colorings. We prove that G has an interval coloring using 6 colors when G is a (3,4)-biregular bigraph having a spanning subgraph whose components are paths with endpoints at 3-valent vertices and lengths in $\{2, 4, 6, 8\}$. We provide sufficient conditions for the existence of such a subgraph.

149. Independence number of 2-factor-plus-triangles graphs (with J. Vandebussche; *Electronic J. Comb.* 16 (2009), Paper #R27, 14 pages). A 2-factor-plus-triangles graph is the union of two 2-regular graphs with the same vertices, one of which consists of disjoint triangles. Let \mathcal{G} be the family of such graphs. These include the famous “cycle-plus-triangles” graphs shown to be 3-choosable by Fleischner and Stiebitz. The independence ratio of a graph in \mathcal{G} may be less than $1/3$; but the only such graphs achieving the minimum value $1/4$ are disjoint unions of copies of one 12-vertex graph. Nevertheless, \mathcal{G} contains infinitely many connected graphs with independence ratio less than $4/15$. Motivated by a question of Erdős, we also construct graphs in \mathcal{G} with girth 7 and independence ratio less than $1/3$ but conjecture that girth 8 guarantees ratio $1/3$. Finally, unions of two graphs whose components have at most s vertices are s -choosable.

148. Implications among linkage properties in graphs (with Q. Liu and G. Yu; *J. Graph Theory* 60 (2009), 327–337).

Given a graph H with vertices w_1, \dots, w_m , a graph G with at least m vertices is H -linked if for every choice of vertices v_1, \dots, v_m in G , there is a subdivision of H in G such that v_i is the branch vertex representing w_i (for all i). This concept generalizes the notions of k -linked, k -connected, and k -ordered graphs. For graphs H_1 and H_2 with the same order that are not contained in stars, the property of being H_1 -linked implies that of being H_2 -linked if and only if $H_2 \subseteq H_1$. The implication also holds when H_1 is obtained from H_2 by replacing an edge

xy with an edge from y to a new vertex x' . Other instances of non-implication are obtained, using a lemma that the number of vertices appearing in minimum vertex covers of a graph G is at most the vertex cover number plus the size of a maximum matching.

147. Repetition number of graphs (with Y. Caro; *Electronic J. Comb.* 16 (2009), Paper #R7, 14 pages). Every n -vertex graph has two vertices with the same degree (if $n \geq 2$). In general, let $\text{rep}(G)$ be the maximum multiplicity of a vertex degree in G . An easy counting argument yields $\text{rep}(G) \geq n/(2d - 2s + 1)$, where d is the average degree and s is the minimum degree of G . Equality can hold when $2d$ is an integer, and the bound is approximately sharp in general, even when G is restricted to be a tree, maximal outerplanar graph, planar triangulation, or claw-free graph. Among large claw-free graphs, repetition number 2 is achievable, but if G is an n -vertex line graph, then $\text{rep}(G) \geq \frac{1}{4}n^{1/3}$. Among line graphs of trees, the minimum repetition number is $\Theta(n^{1/2})$. For line graphs of maximal outerplanar graphs, trees with perfect matchings, or triangulations with 2-factors, the lower bound is linear.

146. Optimal strong parity edge-coloring of complete graphs (with D. Bunde, K. Milans, and H. Wu; *Combinatorica* 23 (2008), 625–632).

Let $\hat{p}(G)$ be the least number of colors in an edge-coloring of G having no open walk along which every color appears on an even number of steps (this is a *strong parity edge-coloring*; see next abstract). We prove that $\hat{p}(K_n) = 2^{\lceil \lg n \rceil} - 1$ for all n . The optimal strong parity edge-coloring of K_n is unique when n is a power of 2, and the optimal colorings are completely described for all n . The proofs use binary vector spaces, and the result strengthens a special case of Yuzvinsky's Theorem.

145. Duality for semiantichains and unichain coverings in products of special posets (with Q. Liu; *Order* 25 (2008), 359–367).

Saks and West conjectured that for every product of partial orders, the maximum size of a semiantichain equals the minimum number of unichains needed to cover the product. We prove the case where both factors have width 2. We also use the characterization of product graphs that are perfect to prove other special cases, including the case where both factors have height 2. Finally, we make an observation about the case where both factors have dimension 2.

144. Triangle-free planar graphs with minimum degree 3 have radius at least 3 (with S.-J. Kim; *Discuss. Math. Graph. Th.* 28 (2008), 563–566). We prove that every triangle-free planar graph with minimum degree 3 has radius at least 3; equivalently, no vertex neighborhood is a dominating set.

143. (5, 2)-coloring of sparse graphs (with O.V. Borodin, S.G. Hartke, A.G. Ivanova, and A.V. Kostochka; *Siberian Electr. Math. Reports* 5 (2008), 417–426).

We prove that every triangle-free graph whose subgraphs all have average degree less than $12/5$ has a (5, 2)-coloring. This includes planar and projective-planar graphs with girth at least 12. Also, the degree result is sharp; we construct a minimal non-(5, 2)-colorable triangle-free graph with 10 vertices that has average degree $12/5$.

142. The hub number of a graph (with T. Grauman, S.G. Hartke, A. Jobson, B. Kinnersley, L. Wiglesworth, P. Worah, and H. Wu; *Info. Proc. Letters* 108 (2008), 226–228). A *hub set* in a graph G is a set $U \subseteq V(G)$ such that any two vertices outside U are connected by a path whose internal vertices lie in U . We prove that $h(G) \leq h_c(G) \leq \gamma_c(G) \leq h(G) + 1$, where $h(G)$, $h_c(G)$, $\gamma_c(G)$ respectively are the minimum sizes of a hub set in G , a hub set inducing a connected subgraph, and a connected dominating set. Furthermore, all graphs with $\gamma_c(G) > h_c(G) \geq 4$ are obtained by substituting graphs into three consecutive vertices of a cycle; this yields a polynomial-time algorithm to check whether $h_c(G) = \gamma_c(G)$.

141. Long local searches for large bipartite subgraphs (with H. Kaul; *SIAM J. Discr. Math.* 22 (2008), 1138–1144).

Given a partition of the vertices of a graph into two sets, a *flip* is a move of a vertex from its own set to the other, under the condition that it has more neighbors in its own set than in the other. A sequence of flips eventually produces a bipartite subgraph capturing more than half of the edges in the graph. Each flip gains at least one

edge. For an n -vertex graph, we show that there is always a maximal flip sequence of length at most $n/2$, and we construct a graph having a flip sequence of length $\frac{2}{25}n^2 + O(n)$.

140. Tree-thickness and caterpillar-thickness under girth constraints (with Q. Liu; *Electronic J. Comb.* 15 (2008), Paper #R93, 11pp).

We study decomposition of connected n -vertex graphs into trees and into caterpillars. The minimum number of such graphs needed is the *tree thickness* or *caterpillar thickness*. The tree thickness of an n -vertex connected graph with girth g is $\lfloor n/g \rfloor + 1$ when $g \geq 5$. Under additional restrictions, the bound also holds for girth 4. For caterpillar thickness, tighter bounds hold for connected n -vertex outerplanar graphs. With triangles forbidden, caterpillar thickness is at most $\lceil 3n/8 \rceil$, which is sharp when $n \equiv 5 \pmod{8}$. With girth at least 6 and $n > 6$, the maximum is $\lceil (n-2)/4 \rceil$. For 2-connected outerplanar graphs with girth g , the maximum is at most $\lfloor n/g \rfloor$ when $n \geq 2g$, with equality when $n \geq g^2/2$.

139. Pebbling and optimal pebbling in graphs (with D.P. Bunde, E.W. Chambers, D. Cranston, and K. Milans; *J. Graph Theory* 57 (2008), 215–238).

Given a distribution of pebbles on the vertices of a graph G , a *pebbling move* takes two pebbles from one vertex and puts one on a neighboring vertex. The *pebbling number* $\Pi(G)$ is the least k such that for every distribution of k pebbles and every vertex r , a pebble can be moved to r . The *optimal pebbling number* $\Pi_{OPT}(G)$ is the least k such that some distribution of k pebbles permits reaching each vertex.

Using new tools (such as the “Squishing” and “Smoothing” Lemmas), we give short proofs of prior results on these parameters for paths, cycles, trees, and hypercubes, a new linear-time algorithm for computing $\Pi(G)$ on trees, and new results on $\Pi_{OPT}(G)$. If G is connected and has n vertices, then $\Pi_{OPT}(G) \leq \lceil 2n/3 \rceil$ (sharp for paths and cycles). Let $a_{n,k}$ be the maximum of $\Pi_{OPT}(G)$ when G is a connected n -vertex graph with $\delta(G) \geq k$. Always $2\lceil \frac{n}{k+1} \rceil \leq a_{n,k} \leq 4\lceil \frac{n}{k+1} \rceil$, with a better lower bound when k is a nontrivial multiple of 3. Better upper bounds hold for n -vertex graphs with minimum degree k having large girth; a special case is $\Pi_{OPT}(G) \leq 16n/(k^2 + 17)$ when G has girth at least 5 and $k \geq 4$. Finally, we compute $\Pi_{OPT}(G)$ in special families such as prisms and Möbius ladders.

138. Circular chromatic index of the Cartesian product of graphs (with X. Zhu; *J. Graph Theory* 57 (2008), 7–18).

The *circular chromatic index* of a graph G , written $\chi'_c(G)$, is the minimum r permitting a function $f: E(G) \rightarrow [0, r)$ such that $1 \leq |f(e) - f(e')| \leq r - 1$ when e and e' are incident. Let H be an $(s-2)$ -regular graph of odd order, where $s \equiv 0 \pmod{4}$. We prove that $\chi'_c(H \square C_{2m+1}) \geq s + 1/\lfloor \lambda(1 - 1/s) \rfloor$, where \square denotes Cartesian product and λ is the minimum of the maximum size of a cycle used in a basis of the cycle space of an orientation of H . When $H = C_{2k+1}$ and m is large, the lower bound is sharp. In particular, if $m \geq 3k + 1$, then $\chi'_c(C_{2k+1} \square C_{2m+1}) = 4 + 1/\lceil 3k/2 \rceil$, independent of m .

137. Parity and strong parity edge-colorings of graphs (with D. Bunde, K. Milans, and H. Wu; *Congr. Numer.* 187 (2007), 193–213).

A *parity walk* in an edge-coloring of a graph is a walk along which each color is used an even number of times. Let $p(G)$ be the least number of colors in a *parity edge-coloring* of G (a coloring having no parity path). Let $\hat{p}(G)$ be the least number of colors in a *strong parity edge-coloring* of G (a coloring having no open parity walk). Always $\hat{p}(G) \geq p(G) \geq \chi'(G)$.

A connected graph G lies in the hypercube Q_k if and only if G has a parity k -edge-coloring in which every cycle is a parity walk. Hence $p(G) \geq \lceil \lg n(G) \rceil$, with equality for paths and even cycles. When n is odd, $p(C_n) = \hat{p}(C_n) = 1 + \lceil \lg n \rceil$. We give examples where $p(G)$ and $\hat{p}(G)$ are equal and examples where they differ; equality is conjectured to hold for all bipartite graphs. The main result that $\hat{p}(K_n) = 2^{\lceil \lg n \rceil} - 1$ for all n will appear in a subsequent paper; we show here that $p(K_n) = \hat{p}(K_n)$ holds for $n \leq 16$ (conjectured for all n). Also, $p(K_{2,n}) = \hat{p}(K_{2,n})$, with value n when n is even and $n + 1$ when n is odd. In general, $\hat{p}(K_{m,n}) \leq m' \lceil n/m' \rceil$, where $m' = 2^{\lceil \lg m \rceil}$.

Let $p_r(G)$ be the least number of colors needed to assign r colors to each edge of G so that every choice of

a color from the list assigned to each edge yields a parity edge-coloring. Trivially, $p_r(G) \leq rp(G)$; we prove that equality holds for paths.

136. Some conjectures of Graffiti.pc on total domination (with E. DeLaVina, Q. Liu, R. Pepper, and B. Waller; *Congr. Numer.* 185 (2007), 81–95).

The *total domination number* of a graph G , written $\gamma_t(G)$, is the minimum size of a set of vertices that intersects the neighborhood of every vertex. We present several upper and lower bounds that originated as conjectures of Graffiti.pc. First, $\gamma_t(G) \geq r$, where r is the radius of G , with equality if and only if every smallest total dominating set induces $r/2$ edges. If μ is the maximum size of a matching in G and ρ is the minimum size of a set of disjoint paths covering the vertex set, then $\gamma_t \leq \mu + \rho$. If G is a connected 3-regular graph, then $\gamma_t(G) \geq 2\rho$. For a nontrivial tree, the total domination number is greater than the minimum size of a maximal matching.

135. Bounds for cut-and-paste sorting of permutations (with D. Cranston and I.H. Sudborough; *Discrete Math.* 307 (2007), 2866–2870).

We consider the problem of determining the maximum number of moves required to sort a permutation of $[n]$ using cut-and-paste operations, in which a segment is cut out and then pasted into the remaining string, possibly reversed. We give short proofs that every permutation of $[n]$ can be transformed to the identity in at most $\lfloor 2n/3 \rfloor$ such moves and that some permutations require at least $\lfloor n/2 \rfloor$ moves.

134. Improved bounds on families with restricted k -wise set intersections (with W.-T. Cao and K.-W. Hwang; *Graphs and Combin.* 23 (2007), 381–386).

Let p be a prime, and let L be a set of s congruence classes modulo p . Let \mathcal{H} be a family of subsets of $[n]$ such that the size modulo p of each member of \mathcal{H} is not in L , but the size modulo p of every intersection of k distinct members of \mathcal{H} is in L . We prove that $|\mathcal{H}| \leq (k-1) \sum_{i=0}^s \binom{n-1}{i}$, improving the bound due to Grolmusz.

133. Extending precolorings to circular colorings (with M.O. Albertson; *J. Comb. Theory (B)* 96 (2006), 472–481).

A (k, d) -coloring of a graph G assigns each vertex a value in \mathbb{Z}_k so that the labels on adjacent vertices differ by at least d . Fix positive integers k', d', k, d such that $k'/d' > k/d \geq 2$. If P is a set of vertices in a (k, d) -colorable graph G , and vertices of P are separated by pairwise distance at least $2 \lceil \frac{kk'}{2(k'd - kd')} \rceil$, then every coloring of P with colors in the set of integers modulo k' extends to a (k', d') -coloring of G . If $k'd - kd' = 1$ (or $k/d = 2$), then this distance threshold is nearly sharp.

132. Chvátal’s condition cannot hold for a graph and its complement (With A.V. Kostochka; *Discuss. Math. Graph. Th.* 26 (2006), 73–76).

Chvátal’s Condition is a sufficient condition for a spanning cycle in an n -vertex graph. The condition is that when the vertex degrees are d_1, \dots, d_n in nondecreasing order, $i < n/2$ implies that $d_i > i$ or $d_{n-i} \geq n - i$. We prove that this condition cannot hold in both a graph and its complement, and we raise the problem of finding its asymptotic probability in the random graph with edge probability $1/2$.

131. Nordhaus–Gaddum-type theorems for decompositions into many parts (with Z. Füredi, A.V. Kostochka, R. Škrekovski, and M. Stiebitz; *J. Graph Theory* 50 (2005), 273–292).

A k -decomposition (G_1, \dots, G_k) of a graph G is a partition of its edge set to form k spanning subgraphs G_1, \dots, G_k . The classical theorem of Nordhaus and Gaddum bounds $\chi(G_1) + \chi(G_2)$ and $\chi(G_1)\chi(G_2)$ over all 2-decompositions of K_n . For a graph parameter p , let $p(k; G)$ denote the maximum of $\sum_{i=1}^k p(G_i)$ over all k -decompositions of the graph G .

The clique number ω , chromatic number χ , list chromatic number χ_l , and Szekeres–Wilf number σ satisfy $\omega(2; K_n) = \chi(2; K_n) = \chi_l(2; K_n) = \sigma(2; K_n) = n + 1$. We obtain lower and upper bounds for $\omega(k; K_n)$, $\chi(k; K_n)$, $\chi_l(k; K_n)$, and $\sigma(k; K_n)$. The last three behave differently for large k . We also obtain lower and upper bounds for the maximum of $\chi(k; G)$ over all graphs embedded on a given surface.

130. Hypergraph extension of the Alon-Tarsi List-Coloring Theorem (with R. Ramamurthi; *Combinatorica* 25 (2005), 355–366).

Alon and Tarsi proved that a graph is d -choosable when it has an orientation that has no vertex of outdegree at least d and has the property that the number of Eulerian subgraphs with an even number of edges differs from the number with an odd number of edges. We generalize this theorem to k -uniform hypergraphs, where k is prime. We use a concept of hypergraph orientation in which a source is chosen from each edge. The generalization of Eulerian subgraph is called “balanced partition” of the edges.

129. Precoloring extensions of Brooks’ Theorem (with M.O. Albertson and A.V. Kostochka; *SIAM J. Discr. Math.* 18 (2004), 542–553).

Let G be a connected graph with maximum degree k (other than a complete graph or odd cycle), let W be a pre-colored set of vertices in G inducing a subgraph F , and let D be the minimum distance in G between components of F . If the components of F are complete graphs and $D \geq 8$ (for $k \geq 4$) or $D \geq 10$ (for $k = 3$), then every proper k -coloring of F extends to a proper k -coloring of G . If the components of F are single vertices and $D \geq 8$, and the vertices outside W are assigned color lists of size k , then every k -coloring of F extends to a proper coloring of G with the color on each vertex chosen from its list. These results are sharp.

128. The visibility number of a graph (with Y.-W. Chang, J. Hutchinson, M.S. Jacobson, and J. Lehel; *SIAM J. Discr. Math.* 18 (2004), 462–471).

Assign to each vertex of a graph G a union of horizontal intervals (“bars”) in the plane. This is a *bar visibility representation* of G if $uv \in E(G)$ precisely when some bar for u is visible from some bar for v via a vertical segment intersecting no bar. The *bar visibility number* $b(G)$ is the minimum t such that G has such a representation assigning at most t bars to each vertex. Results:

- 0) $b(G) \geq (e + 6)/(3n)$ for graphs with n vertices and e edges.
- 1) Every planar graph has bar visibility number at most 2, which is sharp.
- 2) $r \leq b(K_{m,n}) \leq r + 1$, where $r = \lceil \frac{mn+4}{2m+2n} \rceil$.
- 3) $b(K_n) = \lceil n/6 \rceil$.
- 4) $b(G) \leq \lceil n/6 \rceil + 2$ for every n -vertex graph G , using Lovász’s result that graphs with m vertices decompose into $\lfloor m/2 \rfloor$ paths and cycles.

127. Pattern Ramsey numbers (with R.E. Jamison; *Graphs and Combin.* 20 (2004), 333–339).

A *color pattern* is a graph whose edges have been partitioned into color classes. A family \mathcal{F} of color patterns is a *Ramsey family* if there is some integer N such that every edge-coloring of K_N has a copy of some pattern in \mathcal{F} . The smallest such N is the *Ramsey number* $R(\mathcal{F})$ of \mathcal{F} . The classical Canonical Ramsey Theorem of Erdős and Rado yields an easy characterization of the Ramsey families of color patterns.

In this paper we determine $R(\mathcal{F})$ for all families consisting of equipartitioned stars, and we prove that $5\lfloor \frac{s-1}{2} \rfloor + 1 \leq R(\mathcal{F}) \leq 3s - \sqrt{3s}$ when \mathcal{F} consists of a monochromatic star of size s and a polychromatic triangle.

126. Interval numbers of powers of block graphs (with M. Chen and G.J. Chang; *Discrete Math.* 275 (2004), 87–96).

The *interval number* of a graph G is the minimum t such that G is the intersection graph of a family of unions of at most t intervals on the real line. A *block graph* is the intersection graph of the block in some graph; these are known to be the graphs in which every block is a complete subgraphs. The k th *power* of a graph G is the graph with vertex set $V(G)$ in which vertices are adjacent if and only if their distance in G is at most k .

We study interval numbers of powers of block graphs, proving that the interval number of the k th power of a block graph is at most $k + 1$. We also characterize block graphs whose k th powers are interval graphs. As a graph is its first power and trees are block graphs, these results generalize Trotter and Harary’s results that the interval number of a tree is at most two, and a tree is an interval graph if and only if it is a caterpillar.

125. Homomorphisms from sparse graphs with large girth (with O.V. Borodin, S.-J. Kim, and A.V. Kostochka; *J. Comb. Theory (B)* 90 (2004), 147–159).

A (k, d) -*coloring* of a graph G is a map from $V(G)$ to \mathbb{Z}_k such that the images of adjacent vertices differ by at least d in \mathbb{Z}_k . The circular chromatic number of G is the minimum k/d such that G has a (k, d) -coloring. A

homomorphism of G into H is an edge-preserving function from $V(G)$ to $V(H)$.

We show that a planar graph with girth at least $\frac{20t-2}{3}$ has circular chromatic number at most $2 + \frac{1}{t}$, improving earlier results. This follows from a general result establishing homomorphisms into special targets for graphs with given girth and given maximum average degree. Other applications concern oriented chromatic number and homomorphisms into mixed graphs with colored edges.

124. Graphic and protographic lists of integers (with D. Fon-Der-Flaass; *Electronic J. Comb.* 11 (2004), paper R4 (electronic), 5 pages).

A positive list (list of positive integers) is *protographic* if its merger with all but finitely many positive graphic lists is graphic. Define the family \mathcal{P}_s of *s-protographic* lists by letting \mathcal{P}_0 be the family of positive graphic lists and letting \mathcal{P}_s for $s > 0$ be the family of positive lists whose merger with all but finitely many lists in \mathcal{P}_{s-1} is in \mathcal{P}_{s-1} .

The main result is that $X \in \mathcal{P}_s$ if and only if $t(X) \in \mathcal{P}_{s-1}$, where $t(X)$ is the list obtained from X by subtracting one from each term of X (deleting those that become 0) and appending a 1 for each term of X . A corollary is that the maximum number of iterations to reach a graphic list from an n -term even list with sum $2k$ is $k - n + 1$ (when $k \geq n$), achieved by the unique such list having one term larger than 1.

123. Maximum face-constrained coloring of plane graphs (with R. Ramamurthi; *Discrete Math.* 274 (2004), 233–240, and *Electr. Notes in Discrete Math.* Volume 11 (July 2002 online publication from Proc. 9th Intl. Graph Theory Conf. Kalamazoo 2000)).

For a simple connected plane graph G , let $f(G)$ be the maximum number of colors in a vertex coloring of G such that every face is incident to at least two vertices of the same color. If G has n vertices and chromatic number k , then $f(G) \geq \lceil n/k \rceil + 1$. For $k \in \{2, 3\}$, this bound is sharp. For $k = 4$, the optimal bound may be larger by 1.

122. Edge-colorings of complete graphs that avoid polychromatic trees (with T. Jiang); *Discrete Math.* 274 (2004), 137–145, and *Electr. Notes in Discrete Math.* Volume 11 (July 2002 online publication from Proc. 9th Intl. Graph Theory Conf. Kalamazoo 2000)).

In an edge-coloring of a complete graph, a *polychromatic* subgraph is one whose edges receive distinct colors. We study the maximum number of colors in edge-colorings of the n -clique without polychromatic trees of size k . We solve the problem completely when all polychromatic trees of size k are forbidden. When a single tree is forbidden, we determine the answer asymptotically for tree of various types. It turns out that forbidding a particular polychromatic broom is almost as restrictive as forbidding all polychromatic trees of that size.

121. Probabilistic methods for decomposition dimension of graphs (with M. Hagita and A. Kündgen; *Graphs and Combin.* 19 (2003), 493–503).

Given an edge-coloring of a graph, associate a vector with each edge e by letting the i th coordinate be the distance from e to a nearest edge of color i . Chartrand, Erwin, Raines, and Zhang defined the *decomposition dimension* of a graph to be the smallest number of colors in an edge-coloring such that the associated vectors are distinct. For the complete graph with n vertices, they proved that the decomposition dimension is at most $(2n+5)/3$. We show that the decomposition dimension of the complete graph on n vertices is between $(2 - o(1)) \lg n$ and $(3.2 + o(1)) \lg n$. For the k -dimensional hypercube, we give a lower bound of $k/\lg k$ and an upper bound of $(3.17 + o(1))k/\lg k$. For “nearly” regular graphs with bounded diameter, we show that decomposition is well-behaved in a precise sense.

120. A list analogue of equitable coloring (with A.V. Kostochka and M.J. Pelsmayer; *J. Graph Theory* 44 (2003), 166–177).

Given lists of available colors assigned to the vertices of a graph G , a *list coloring* is a proper coloring of G such that the color on each vertex is chosen from its list. If the lists all have size k , then a list coloring is *equitable* if each color appears on at most $\lceil n(G)/k \rceil$ vertices. A graph is *equitably k-choosable* if such a coloring exists whenever the lists all have size k .

We prove that G is equitably k -choosable when $k \geq \max\{\Delta(G), n(G)/2\}$ unless G contains K_{k+1} or k is odd and $G = K_{k,k}$. For forests, the threshold improves to $k \geq 1 + \Delta(G)/2$. If G is a 2-degenerate graph (given $k \geq 5$) or a connected interval graph (other than K_{k+1}), then G is equitably k -choosable when $k \geq \Delta(G)$.

119. On the Erdős–Simonovits–Sós Conjecture about the anti-Ramsey number of a cycle (with T. Jiang; *Combinatorics, Probability, and Computing* 12 (2003), 585–598).

Given a positive integer n and a family \mathcal{F} of graphs, let $f(n, \mathcal{F})$ denote the maximum number of colors in an edge-coloring of K_n such that no subgraph of K_n belonging to \mathcal{F} has distinct colors on its edges. Erdős, Simonovits, and Sós conjectured for fixed $k \geq 3$ that $f(n, \{C_k\}) \in \left(\frac{k-2}{2} + \frac{2}{k-1}\right)n + O(1)$ and provided a construction achieving this bound. For general k , we improve the previous bound of $(k-2)n - \binom{k-1}{2}$ to $f(n, \{C_k\}) \leq \left(\frac{k+1}{2} - \frac{2}{k-1}\right)n - (k-2)$. For even k , we further improve it to $(k/2)n - (k-2)$.

118. Isometric cycles and bridged graphs (with T. Jiang and S.-J. Kim; *J. Graph Theory* 43(2003), 161–170).

A graph G is *bridged* if every cycle C of length at least 4 has two vertices x and y such that $d_G(x, y) < d_C(x, y)$. We show that every minimal cutset S in a bridged graph G induces a connected subgraph of G . We also construct examples showing that for every connected simple graph H with girth at least 6 (including trees), there exists a bridged graph G such that G has a unique minimum cutset S and that $G[S] = H$. This provides counterexamples to Hahn’s conjecture that $d_G(u, v) \leq 2$ when u and v lie in a minimum cutset in a bridged graph G . We also consider the convexity of cutsets in bridged graphs.

117. On the existence of Hamiltonian paths in the cover graph of $M(n)$ (with C.D. Savage and I. Shields; *Discrete Math.* 262(2003), 241–252).

The poset $M(n)$ has as its elements the nondecreasing n -tuples of nonnegative integers whose nonzero entries are distinct, with $a \leq b$ if $a_i \leq b_i$ for $1 \leq i \leq n$. We prove that the cover graph of $M(n)$ has a Hamiltonian path if and only if $\binom{n+1}{2}$ is odd and $n \neq 5$.

116. Restricted edge-colorings of bicliques (with D. Mubayi; *Discrete Math.* 257(2002), 513–529).

We investigate edge-colorings of $K_{n,n}$ such that every C_4 receives at least q , and at most q' colors, where $1 \leq q \leq q' \leq 4$. In many cases we determine both the maximum and the minimum number of colors in such a coloring.

115. Chromatic spectrum of mixed hypergraphs (with T. Jiang, D. Mubayi, Zs. Tuza, and V.I. Voloshin; *Graphs and Combin.* 18(2002), 309–318).

A *mixed hypergraph* is a triple $H = (X, \mathcal{C}, \mathcal{D})$, where X is the *vertex set* and each of \mathcal{C} , \mathcal{D} is a list of subsets of X . A *strict k -coloring* of H is a surjection $c : X \rightarrow [k]$ such that each member of \mathcal{C} has two vertices assigned a common value and each member of \mathcal{D} has two vertices assigned distinct values. The feasible set of H is the set of integers k for which H has a strict k -coloring. We determine which finite sets can be feasible sets of mixed hypergraphs. For the set $\{s, t\}$ with $2 \leq s \leq t - 2$, the smallest realization has $2t - s$ vertices. We also provide other efficient constructions and show that gaps can arise in the feasible sets for r -uniform mixed hypergraphs.

114. A Fibonacci tiling of the plane (with C.W. Huegy; *Discrete Math.* 249(2002), 111–116).

We describe a tiling of the plane, motivated by geodesic domes, that involves the Fibonacci series in many ways.

113. A Proof of the two-path conjecture (with H.J. Fleischner, R.R. Molina, and K.W. Smith; *Electronic J. Combinatorics* 9(2002), 3 pages).

Let G be a connected graph that is the edge-disjoint union of two paths of length n , where $n \geq 2$. Using a result of Thomason on decompositions of 4-regular graphs into pairs of Hamiltonian cycles, we prove that G has a third path of length n .

112. Cevian dissections of a triangle (with V.J. Matsko and J.E. Wetzel; *J. of Geometry* 72(2001), 115–127).

We determine the possible numbers of regions, segments, and intersection points that can arise when p , q , and r segments are drawn from the three vertices of a triangle, respectively, to the opposite sides.

111. Structural diagnosis of wiring networks: finding connected components of an unknown subgraph (with W. Shi; *SIAM J. Discr. Math.* 14(2001), 510–523).

This paper is a slight elaboration of #83. We study the diagnosis of short faults among nets. The corresponding graph problem is to the connection classes of an unknown fault subgraph H of a graph G of potential faults. A

query is a vertex subset; the oracle returns all vertices connected in H to the query set. The *query number* of G is the worst-case minimum number of queries to compute the connection classes of H . The *test number* is the complexity for non-adaptive algorithms.

We present a general method for upper bounds on these parameters and use it to compute values in some special classes of graphs. Adaptive diagnosis often uses exponentially fewer tests than traditional non-adaptive diagnosis, and diagnosis using information about G may use dramatically fewer tests than the traditional assumption that G is a complete graph.

110. Realizing degree imbalances in directed graphs (with D. Mubayi and T.G. Will; *Discrete Math.* 239(2001), 147–153).

In a directed graph, the *imbalance* of a vertex v is $b(v) = d^+(v) - d^-(v)$. We provide a necessary and sufficient condition for a sequence of n integers to be realized as the sequence of imbalances of a simple directed graph on n vertices. Moreover, a realization of a realizable sequence can be produced by a greedy algorithm.

109. Ramsey theory and bandwidth of graphs (with Z. Füredi; *Graphs and Combin.* 17(2001), 147–153). The bandwidth $B(G)$ of a graph G is the minimum, over all labelings of the vertices with distinct integers, of the maximum difference between labels of adjacent vertices. Let $f(n)$ denote the maximum sum of the bandwidths of complementary n -vertex graphs. Chinn, Chung, Erdős, and Graham [1981] proved the existence of constants c_1, c_2 such that $2n - c_2 \log n \leq f(n) \leq 2n - c_1 \log n$. We sharpen this to $2n - 8 \log_2 n - O(1) \leq f(n) \leq 2n - 4 \log_2 n + o(\log n)$.

108. On the number of vertices with specified eccentricity (with D. Mubayi; *Graphs and Combin.* 16(2000), 441–452).

The *eccentricity* of a vertex v in a graph is the maximum of the distances from v to all other vertices. The *diameter* of a graph is the maximum of the eccentricities of its vertices. Fix the parameters n, d, c . Over all graphs with order n and diameter d , we study the minimum and maximum of the number of vertices with eccentricity c .

107. Edge-bandwidth of theta graphs (with D. Eichhorn, D. Mubayi, and K. O’Byrant; *J. Graph Theory* 35(2000), 89–98).

An *edge-labeling* f of a graph G is an injection from $E(G)$ to the set of integers. The *edge-bandwidth* of G is $B'(G) = \min_f \{B'(f)\}$, where $B'(f)$ is the maximum difference between labels of incident edges of G . The *m -theta graph* $\Theta(l_1, \dots, l_m)$ is the graph consisting of m pairwise internally disjoint paths with common endpoints and lengths $l_1 \leq \dots \leq l_m$. We determine the edge-bandwidth of all m -theta graphs.

106. Multiple vertex coverings by specified induced subgraphs (with Z. Füredi and D. Mubayi; *J. Graph Theory* 34(2000), 180–190).

Given graphs H_1, \dots, H_k , let $f(H_1, \dots, H_k)$ be the minimum order of a graph G such that for each i , the induced copies of H_i in G cover $V(G)$. We prove constructively that $f(H_1, H_2) \leq 2(n(H_1) + n(H_2) - 2)$; equality holds when $H_1 = \overline{H_2} = K_n$. We prove that $f(H_1, \overline{K_n}) = n + 2\sqrt{\delta(H_1)n} + O(1)$ as $n \rightarrow \infty$ and determine $f(K_{1,m-1}, \overline{K_n})$ exactly.

105. Connected domination and spanning trees with many leaves (with Y. Caro and R. Yuster; *SIAM J. Discr. Math.* 13(2000), 202–211).

Let $G = (V, E)$ be a connected graph. A *connected dominating set* $S \subset V$ is a dominating set that induces a connected subgraph of G . The *connected domination number* of G , denoted $\gamma_c(G)$, is the minimum cardinality of a connected dominating set. Alternatively, $|V| - \gamma_c(G)$ is the maximum number of leaves in a spanning tree of G . Let δ denote the minimum degree of G . We prove that $\gamma_c(G) \leq |V| \frac{\ln(\delta+1)}{\delta+1} (1 + o_\delta(1))$. Two algorithms that construct a set this good are presented. One is a sequential polynomial time algorithm, while the other is a randomized parallel algorithm in RNC.

104. Perfection thickness of graphs (with H. Asari, T. Jiang, and A. Kündgen); *Discrete Math.* 215(2000), 263–264).

We determine the order of growth of the worst-case number of perfect subgraphs needed to cover an n -vertex graph.

103. Generalized chromatic number and generalized girth (with B. Bollobás; *Discrete Math.* 213(2000), 29–34).

Erdős proved that there are graphs with arbitrarily large girth and chromatic number. We study the extension of this for generalized chromatic numbers. Let H' be the class of graphs that do not contain H as a subgraph. Let $\chi_{H'}(G)$ denote the number of colors needed to label the vertices of G so that the subgraph induced by each color class does not contain H . For $j \geq 2$, an (H, j) -cycle in a graph G is a list of distinct subgraphs H_1, \dots, H_j , each isomorphic to H , such that $\bigcup_{i=1}^j H_i$ contains a cycle that decomposes into j nontrivial paths with the i th path in H_i (any two successive paths in the decomposition share one vertex). The H -girth of G , written $g_H(G)$, is the minimum j such that G contains an (H, j) -cycle, if this exists; otherwise $g_H(G) = \infty$. Theorem: If s, k are positive integers with $s > r$, then there is a graph G with $g_H(G) > s$ and $\chi_{H'}(G) > k$. Furthermore, if n is sufficiently large, then there is a graph G of order n with $g_H(G) > s$ and $\chi_{H'}(G) \geq n^{1/(r^\varepsilon s)}$, where $\varepsilon = 1$ if H is 2-connected and $\varepsilon = 2$ if H is not 2-connected.

102. Partially Ordered Sets (with G. Brightwell; Chapter 11 in *Handbook of Discrete and Combinatorial Mathematics* (K.H. Rosen, editor-in-chief), (CRC Press, 2000), 717–752).

An invited expository summary of fundamental results about partially ordered sets.

101. Every outerplanar graph is the union of two interval graphs (with A.V. Kostochka; *Proc. 30th S.E. Intl. Conf. Graph Th. Comb. Comp. Congr. Numer.*139(1999), 5–8).

It has long been known that every outerplanar graph has interval number at most two. We strengthen this by proving that every outerplanar graph is the union of two interval graphs. Thus an interval representation can be formed with each vertex allowed one interval in each of two separate “tracks”.

100. Edge-bandwidth of graphs (with T. Jiang, D. Mubayi, and A. Shastri; *SIAM J. Discr. Math.* 12(1999), 307-316).

The *edge-bandwidth* of a graph is the minimum, over all labelings of the edges with distinct integers, of the maximum difference between labels of two incident edges. We prove that edge-bandwidth is at least as large as bandwidth for every graph, with equality for certain caterpillars. We obtain sharp or nearly-sharp bounds on the change in edge-bandwidth under addition, subdivision, or contraction of edges. We compute edge-bandwidth for K_n , $K_{n,n}$, caterpillars, and some theta graphs.

99. Coloring trees with minimum sum of colors (with T. Jiang; *J. Graph Theory* 32(1999), 354–358).

The *chromatic sum* $\Sigma(G)$ of a graph G is the least sum of labels among all proper colorings with natural numbers. The *strength* $s(G)$ of G is the minimum number of colors needed to achieve the chromatic sum. We construct for each integer k a tree T_k with strength k that has maximum degree only $2k - 2$; this is best possible.

98. Intersection representation of digraphs in trees with few leaves (with I.-J. Lin and M.K. Sen; *J. Graph Theory* 32(1999), 340–353).

Every digraph D has an intersection representation in which each vertex u is assigned a source subtree S_u and a sink subtree T_u in a host tree, with $uv \in E(D)$ if and only if $S_u \cap T_v \neq \emptyset$. If the sink subtrees are single vertices, this is a *catch subtree representation*. The *leafage* $l(D)$ is the minimum number of leaves in a host tree in which D has a subtree intersection representation. Similarly, the *catch leafage* $l^*(D)$ is the minimum number of leaves in a host tree where D has a catch subtree representation. We prove $f(D) \leq f^*(D) \leq l(D) \leq l^*(D) \leq w(P(D)) \leq n$ for every n -vertex digraph. Here $f(D)$ is the Ferrers dimension of D , meaning the minimum number of Ferrers digraphs whose union is \overline{D} , $f^*(D)$ is the minimum number of pairwise-disjoint Ferrers digraphs whose union is \overline{D} , and $w(P(D))$ is the width of the poset of predecessor sets of vertices, ordered by inclusion. Furthermore, there are digraphs for which equality holds throughout this string of inequalities, and for each inequality there are graphs for which the lesser value is constant and the higher value is arbitrarily large.

97. A short proof that “proper = unit” (with K.P. Bogart; *Discrete Math.* 201(1999), 21–23).

A short proof is given that the graphs with proper interval representations are the same as the graphs with unit interval representations.

96. Diagnosis of wiring networks: An optimal randomized algorithm for finding connected components of unknown graphs (with W. Shi; *SIAM J. Computing* 28(1999), 1541–1551).

[This paper is an updated version of #75.]

95. Rectangle number of cubes and complete multipartite graphs (with Y.-W. Chang; *Proc. 29th S.E. Intl. Conf. Graph Th. Comb. Comp.* 132(1998), 19–28).

The *rectangle number* of a graph G is the minimum t such that G is the intersection graph of sets consisting of the union of at most t rectangles in the plane (with vertical and horizontal sides). The rectangle number of a complete multipartite graph is at most 2, with equality if and only if G has at least three partite sets of size at least 2. The rectangle number of the k -dimensional cube is at most $\lceil k/4 \rceil$, except that when $k = 4$ the rectangle number is 2.

94. The leafage of a chordal graph (with I.-J. Lin and T.A. McKee; *Discuss. Math. Graph. Th.* 18(1998), 23–48.)

Every chordal graph is the intersection graph of collection of subtrees of a host tree. The *leafage* of a chordal graph is the minimum number of leaves of the host tree in such a representation. We provide bounds on the leafage and conditions for equality. The leafage is at least the size of the largest *asteroidal set* of vertices, defined to be a set $S \subseteq V(G)$ such that for any $x, y, z \in S$, there is an x, y -path avoiding the neighborhood of z . This bound is tight for k -trees and Husimi trees. The leafage is at most the width of the poset of neighborhoods of simplicial vertices, ordered by inclusion. From this the maximum leafage of an n -vertex chordal graph is the maximum k such that $k \leq \binom{n-k}{\lfloor (n-k)/2 \rfloor}$, which is $n - \lg n - \frac{1}{2} \lg \lg n + O(1)$. We obtain algorithms that compute leafage for special classes of chordal graphs. Finally, we consider analogous questions for the *proper leafage* of a chordal graph, which is the minimum number of leaf of the host tree in a representation in which no assigned subtree contains another.

93. Largest regular graphs with equal connectivity and independence number (with P.K. Kwok; *Proc. 8th Intl. Conf. Graph Theory (Kalamazoo 1996)* (Wiley, 1998), 587–589).

We determine the maximum order of a k -regular graph with independence number equal to connectivity. The answer is $k^2 + 1$ if $k \leq 2$, k^2 if $k \geq 3$ and k is even, and $k^2 - 1$ if $k \geq 3$ and k is odd. achieved when connectivity and independence number both equal k .

92. Line digraphs and coreflexive vertex sets (with X. Liu; *Discrete Math.* 188(1998), 269–277).

A nontrivial *coreflexive set* or *coreset* in a digraph is a set U of vertices such that the set of all predecessors of all successors of U is U itself — that is, $U = N^-(N^+(U))$. Also, the set of all sinks is considered a trivial coreset.

The coresets of a digraph partition its vertex set. This generates a natural *core decomposition* of the edge set, grouping edges by which coreset contains their tails. This leads to characterizations of line digraphs; for example, a digraph is a line digraph if and only if each member of the core decomposition is a cartesian product.

The *coreset digraph* of a digraph is the intersection digraph of the pairs $(U, N^+(U))$, where U is a coreset. In contrast to taking line digraphs, iteration of the coreset digraph operation always converges. The digraphs unchanged under this operation are only the paths and the digraphs consisting of one path emerging from a cycle.

91. Star-factors of tournaments (with G. Chen and X. Lu; *J. Graph Theory* 28(1998), 141–145).

Let S_k denote the m -vertex simple digraph consisting of $m - 1$ edges with a common tail. Let $f(m)$ denote the minimum n such that every n -vertex tournament contains a spanning subgraph consisting of n/m disjoint copies of S_m . We prove that $m \lg m - m \lg \lg m \leq f(m) \leq 4m^2 - 6m$ for sufficiently large m .

90. Bandwidth and density for block graphs (with L.T.Q. Hung, M.L. Weaver, and M.M. Syslo); *Discrete Math.* 189(1998), 163–176.

The *bandwidth* of an n -vertex graph G is the minimum of the maximum difference between adjacent labels when the vertices are labeled with distinct integers. A lower bound (the “local density bound”) is obtained by maximizing, over all subgraphs H of G , the quantity $\lceil (|V(H)| - 1)/\text{diam}(H) \rceil$. Syslo and Zak proved that this bound is optimal

for caterpillars (trees having a single path incident to every edge) and gave an algorithm to obtain a representation satisfying the bound. We extend this result to a class of Husimi trees. A *Husimi tree* is a graph in which every block is a clique. A *Husimi caterpillar* is a Husimi tree in which, if the vertices of degree 1 are deleted, the cut vertices of the remaining graph induce a path. We provide an algorithm to produce a representation achieving the local density bound for graphs in this class. The result is best possible in several respects. We also prove that the bandwidth problem is NP-complete for tolerance graphs and for Husimi trees that are Husimi caterpillars with a pendant edge possibly attached at leaves.

89. Interval number and boxicity of digraphs (with Y.-W. Chang; *Proc. 8th Intl. Conf. Graph Theory* (accepted) - 9 pages).

An intersection representation of a digraph assigns a source set S_v and a sink set T_v to each vertex v such that $u \rightarrow v$ if and only if $S_u \cap T_v \neq \emptyset$. The *interval number* of a digraph D is the minimum t such that D has such a representation with each source set and sink set being the union of at most t intervals on the real line. The interval number is bounded by the maximum in-degree and by the maximum out-degree, and this is best possible. Every n -vertex digraph has interval number at most $n/(\lg n + 1)$, and the interval number of the random digraph is at least $n/(4 \lg(2n))$. For boxicity, we use d -dimensional boxes as source sets and sink sets in the representation and seek to minimize d : the maximum boxicity of an n -vertex digraph is $\lceil n/2 \rceil$.

88. The bricklayer problem and the strong cycle lemma (with H. Snevily; *Amer. Math. Monthly* 105(1998), 131–143).

We consider bricks of length $q + 1$. Atop two contiguous bricks we can place a brick in one of q positions, covering a positive integer length of each brick below it. We count the stacks of bricks buildable on a base of length m by establishing a bijection to the set of 0,1-sequences of length m in which every prefix has at least q times as many 0's as 1. These leads to a formula by using the Cycle Lemma of Dvoretzky and Motzkin.

Kierstead and Trotter extended the Cycle Lemma by giving a combinatorial meaning to each 0 in a circular arrangement of $k + 1$ 0's and k 1's. We extend their result slightly and present various applications, many proved earlier by other means. A binary string is q -good if the number of 0's in it is more than q times the number of 1's. A 0-linearization of a circular arrangement a is a breaking of a to produce a linear string ending at a 0. Suppose a is a circular arrangement of k 1's and $qk + 1$ 0's. Suppose S is a set of t 0's in a , and choose $i \in \{1, \dots, t\}$. Then there is a unique 0-linearization of a ending at a position of S in which exactly i of the prefixes ending at elements of S are q -good. A weaker but best-possible result holds when the arrangement has k 1's and $qk + p$ 0's.

87. Short proofs for interval digraphs (*Discrete Math.* 178(1998), 287-292).

We give short proofs of the matrix characterizations of interval digraphs and unit interval digraphs due to Sen, Das, Roy, and West and to Sen and Sanyal, respectively. An *interval digraph* is a digraph representable by assigning a pair of intervals (S_v, T_v) to each vertex so that $u \rightarrow v$ if and only if $S_u \cap T_v \neq \emptyset$. A *unit interval digraph* is one in which this can be done using intervals of the same length. A 0,1-matrix is the adjacency matrix of an interval digraph if and only if its rows and columns can be permuted independently to obtain a matrix in which the 0's can be labeled with R or C such that every position to the right of a 0 labeled R is a 0 labeled R and every position below a 0 labeled C is a 0 labeled C. A 0,1-matrix is the adjacency matrix of a unit interval digraph if in addition to the condition above, every position above a 0 labeled R is a 0 labeled R and every position to the left of a 0 labeled C is a 0 labeled C.

86. Classes of interval digraphs and 0,1-matrices (with I.-J. Lin and M.K. Sen; *Proc. 28th S.E. Intl. Conf. Graph Th. Comb. Comp., Congr. Numer.* 125(1997), 201–209.)

We consider a hierarchy of four classes of interval digraphs. For each class, we provide a forbidden submatrix characterization for membership in the next class; a digraph in the i th class belongs to the next smaller class if and only if its adjacency matrix contains no submatrix in the forbidden collection. The largest class is the class of interval digraphs; the smallest is the class of unit interval digraphs. A byproduct of the refined hierarchy is a shorter proof of the characterization of interval digraphs that are unit interval digraphs.

85. The number of dependent edges in an acyclic orientation (with D.C. Fisher, K. Fraughnaugh, and L. Langley; *J. Comb. Theory (B)* 71(1997), 73–78.)

A *dependent edge* in an acyclic orientation is an edge whose reversal creates a cycle. It is well known that if the chromatic number $\chi(G)$ is less than the girth $g(G)$, then G has an acyclic orientation without dependent edges (indeed, G is the graph of the cover relation of a poset). Edelman showed that if G is connected, then every acyclic orientation of G has at most $e(G) - n(G) + 1$ dependent arcs. We prove his conjecture that if G is connected with chromatic number less than girth, then G has an acyclic orientation with exactly d dependent arcs for all $d \leq e(G) - n(G) + 1$.

84. Optimal structural diagnosis of wiring networks (with W. Shi); *Proc. 27th Intl. Symp. Fault-Tolerant Computing (FTCS-27) - Seattle 1997* (IEEE Press, 1997), 162–191.

See abstract for full version, published as #111 above.

83. Total interval number for graphs with bounded degree (with A. Kostochka; *J. Graph Theory* 25(1997), 79–94).

We prove constructively that the maximum of the total interval number of an n -vertex graph with maximum degree Δ is $(\Delta + 1/\Delta)n/2$, with equality if and only if every component of the graph is $K_{\Delta, \Delta}$. We also obtain bounds on the maximum value for connected n -vertex graphs with maximum degree Δ .

82. The superregular graphs *J. Graph Theory* 23(1996), 289–295.

A graph is *superregular* if it has no vertices, or if it is regular and the subgraphs induced by the neighbors and by the nonneighbors of each vertex are superregular. The superregular graphs are precisely mK_p (disjoint union of isomorphic cliques), $K_m \square K_m$ (cartesian product of two isomorphic cliques), C_5 (the five-cycle), and the complements of these graphs.

81. The total interval number of a graph II: Trees and complexity (with T.M. Kratzke; *SIAM J. Discr. Math.* 9(1996), 339–348.

Given a simple graph G , the *total interval number* $I(G)$ is the minimum of the total number of intervals used in any intersection representation of G using sets that are unions of real intervals. For triangle-free graphs, $I(G) = m + t(G)$, where $m = |E(G)|$ and $t(G)$ is the minimum number of edge-disjoint trails whose union contains an endpoint of every edge. This yields the NP-completeness of recognizing $I(G) = m + 1$, even for triangle-free 3-regular planar graphs. It also leads to a linear-time algorithm to compute $I(G)$ for trees and a characterization of the trees requiring $m + t$ intervals, for fixed t . Further corollaries include the Aigner/Andreae bound of $I(G) \leq \lfloor 5n - 3 \rfloor / 4$ for trees (achieved by a subdivided star), a characterization of the extremal trees, and an alternate proof of the extremal bound $\lfloor (5m + 2) / 4 \rfloor$ for connected graphs.

80. Large $2P_3$ -free graphs with bounded degree (with M.-S. Chung; *Discrete Math.* 150(1996), 69–79.

Let $ex^*(D; H)$ be the maximum number of edges in a connected graph with maximum degree D and no induced subgraph H ; this is finite if and only if H is a disjoint union of paths. We give a recursive upper bound applicable when H has more than one component. In particular, if the largest component of H has order m , then $ex^*(D; H) = O(D^2 ex^*(D; P_m))$. Furthermore, we prove constructively that $ex^*(D; qP_m) = \Theta(D^2 ex^*(D; P_m))$. Finally, we solve the problem completely for $H = 2P_3$ (except for small values of D), determining the unique extremal graph. The maximum number of edges is $\frac{1}{8}[D^4 + D^3 + D^2 + 6D]$ if D is even and $\frac{1}{8}[D^4 + D^3 + 2D^2 + 3D + 1]$ if D is odd.

79. The path spectrum of a graph (with M.S. Jacobson, A.E. Kézdy, E. Kubicka, G. Kubicki, J. Lehel, and C. Wang; *Congr. Numer.* 112(1995), 49–64).

The *path spectrum* $PS(G)$ of a graph G is the set of lengths of maximal paths in G . We prove several results about $PS(G)$ for graphs and for trees. Determining whether an input graph G has a maximal path of a specified input length is NP-hard, but the complexity of determining whether an input set of integers is realizable as a path spectrum is unknown.

78. Multitrack interval number (with A. Gyárfás; *Congr. Numer.* 109(1995), 109–116).

A *d-track interval* is a union of d intervals, one each from d parallel lines. The intersection graphs of d -track

intervals are the unions of d interval graphs. The *multitrack interval number* or simply *track number* of a graph G is the minimum number of interval graphs whose union is G . We determine the track number for $K_{m,n}$ by proving that the arboricity of $K_{m,n}$ equals its “caterpillar arboricity”. We prove that recognition of graphs with track number 2 is NP-complete.

77. Representing digraphs using intervals or circular arcs (with M.K. Sen and B.K. Sanyal; *Discrete Math.* 147(1995), 235–245).

Containment and overlap representations of digraphs are studied. The interval containment digraphs are the digraphs of Ferrers dimension 2, and the circular arc containment digraphs are the complements of circular arc intersection digraphs. A poset is an interval containment poset if and only if its comparability digraph is an interval containment digraph, and a graph is an interval graph if and only if the corresponding symmetric digraph with loops is an interval digraph. The unit overlap digraphs are the unit interval digraphs, and the adjacency matrices of all overlap digraphs have a simple structural characterization bounding their Ferrers dimension by 3.

76. Optimal algorithms for finding connected components of an unknown graph (with W. Shi; In *Computing and Combinatorics* (1st Intl. Conf. COCOON '95), Xi'an, China, ed. D.-Z. Du and M. Li) *Lecture Notes in Computer Science* 959(1995), 131–140.

We want to find the connected components of an unknown graph G with a known vertex set V . We learn about G by sending an oracle a query set $S \subseteq V$, and the oracle tells us the vertices connected to S . We want to use the minimum number of queries, adaptively, to find the components. The problem is also known as interconnect diagnosis of wiring networks in VLSI. The graph has n vertices and k components, but k is not part of the input. We present a deterministic algorithm using $O(\min\{k, \log n\})$ queries and a randomized algorithm using expected $O(\min\{k, \log k + \log \log n\})$ queries. We also prove matching lower bounds.

75. Interval number of special posets and random posets (with T. Madej; *Discrete Math.* 144(1995), 67–74.

The *interval number* $i(P)$ of a poset P is the smallest t such that P can be represented by containment of sets such that each set $f(x)$ is the union of at most t intervals. For the special poset $B_{n,k}$ consisting of the singletons and k -subsets of an n -element set, ordered by inclusion, $i(B_{n,k}) = \min\{k, n - k + 1\}$ if $|n/2 - k| \geq n/2 - (n/2)^{1/3}$. For general n -element posets of height 1 or posets of height one with n minimal elements, $i(P) \leq \lceil \frac{n}{\lg n - \lg \lg n} \rceil + 1$. Finally, the fraction of the n -element posets having interval number between $(1 - \varepsilon) \frac{n}{8 \lg n}$ and $(3/2) (\lceil \frac{n}{\lg n - \lg \lg n} \rceil + 1)$ approaches 1 as $n \rightarrow \infty$.

74. Parsimonious 2-multigraphs (with T. Will; In *Graph theory, Combinatorics, and Algorithms* (Proc. 7th Intl. Conf. Graph Theory, ed. Y. Alavi and A. Schwenk) (Wiley 1995), 1249–1258.

A 2-multigraph is a loopless multigraph with maximum multiplicity 2; pairs of vertices induce 0, 1, or 2 edges. A 2-multigraph is *parsimonious* if it has the minimum number of single edges (multiplicity 1) among all 2-multigraphs with the same degree sequence. In every parsimonious 2-multigraph, the subgraph of single edges consists of isolated stars and possibly one component that is a triangle. We prove the conjecture of Brualdi and Michael that for any fixed degree sequence, either every parsimonious 2-multigraph with those vertex degrees has a triangle of single edges, or no such parsimonious 2-multigraph has a triangle of single edges.

73. Interval digraphs that are indifference digraphs (with I.-J. Lin; In *Graph theory, Combinatorics, and Algorithms* (Proc. 7th Intl. Conf. Graph Theory, ed. Y. Alavi and A. Schwenk) (Wiley 1995), 751–765.)

A digraph is an *interval digraph* if each vertex can be assigned a source interval and a sink interval on the real line such that there is an edge from u to v if and only if the source interval for u intersects the sink interval for v . A digraph is an *indifference digraph* or *unit interval digraph* if and only if such a representation can be constructed in which every source and sink interval has unit length. We prove that an interval digraph is a unit interval digraph if and only if its adjacency matrix is free of six forbidden submatrices: three 3 by 4 matrices and their transposes.

72. Maximum bandwidth under edge addition (with J.-F. Wang and B. Yao; *J. Graph Theory* 20(1995), 87–90).

We determine how much the bandwidth $B(G)$ of a graph G can increase when a single edge is added. Let $g(b, n)$ be the maximum value of $B(G + e)$ when G has n vertices and bandwidth b . Erdős asked when $B(G + e) \leq B(G) + 1$ holds. We obtain

$$g(b, n) = \begin{cases} b + 1 & \text{if } n \leq 3b + 4 \\ \lceil (n - 1)/3 \rceil & \text{if } 3b + 5 \leq n \leq 6b - 2 \\ 2b & \text{if } n \geq 6b - 1. \end{cases}$$

71. The 2-intersection number of paths and bounded-degree trees (with M.S. Jacobson and A.E. Kézdy; *J. Graph Theory* 19(1995), 461–469).

We represent a graph by assigning each vertex a finite set so that vertices are adjacent if and only if the assigned sets have at least two common elements. The *2-intersection number* $\theta_2(G)$ of a graph G is the minimum size of the union of such sets. We prove that the maximum order of a path with 2-intersection number at most t is between $(t^2 - 19t + 4)/4$ and $(t^2 - t + 6)/4$, so $\theta_2(P_n) \sim 2\sqrt{n}$. We also show the existence of a constant c depending on ϵ such that, for any tree T with maximum degree at most d , $\theta_2(T) \leq c(\sqrt{n})^{1+\epsilon}$. With no bound on maximum degree, there is an n -vertex tree T with $\theta_2(T) > (c'n)^{2/3}$.

70. Gray code enumeration of families of integer partitions (with D. Rasmussen and C.D. Savage; *J. Comb. Theory (A)* 70(1995), 201–229).

The elements of some families of integer partitions can be enumerated in a minimal change, or *Gray code*, order. We construct Gray code enumerations for the classes $P_\delta(n, k)$ and $D(n, k)$ of partitions of n into parts of size at most k in which, for $P_\delta(n, k)$, the parts are congruent to one modulo δ and, for $D(n, k)$, the parts are distinct. The minimal change required between successive partitions is the increase of one part by δ (or the addition of δ ones) and the decrease of one part by δ (or the removal of δ ones), where, in the case of $D(n, k)$, $\delta = 1$.

69. Acyclic orientations of complete bipartite graphs (*Discrete Math.* 138(1995), 393–396).

A *dependent edge* in an acyclic orientation is an edge whose reversal creates a cycle. We characterize the acyclic orientations of complete bipartite graphs and prove that the number of dependent edges can be any integer from $n - 1$ to e , where n and e are the numbers of vertices and edges in the graph. Edelman conjectured this for triangle-free graphs. The proof for graphs with girth exceeding chromatic number appears in #85.

68. The p -intersection number of a complete bipartite graph and orthogonal double coverings of a clique (with M.S. Chung; *Combinatorica* 14(1994), 453–461).

The *p -intersection graph* of a collection of finite sets $\{S_i\}_{i=0}^n$ is the graph with vertices $1, \dots, n$ such that i, j are adjacent if and only if $|S_i \cap S_j| \geq p$. The *p -intersection number* of a graph G , herein denoted $\theta_p(G)$, is the minimum size of a set U such that G is the p -intersection graph of subsets of U . If G is the complete bipartite graph $K_{n,n}$ and $p \geq 2$, then $\theta_p(K_{n,n}) \geq (n^2 + (2p - 1)n)/p$. For $p = 2$, achieving this bound is equivalent to a graph design problem. An *orthogonal double covering* of K_n is a collection of n subgraphs of K_n , each with $n - 1$ edges and maximum degree 2, such that each pair of subgraphs shares exactly one edge. $\theta_2(K_{n,n}) = (n^2 + 3n)/2$ if and only if K_n has an orthogonal double covering. By construction, K_n has an orthogonal double covering whenever n is an odd number that is not a multiple of 3, proving $\theta_2(K_{n,n}) = (n^2 + 3n)/2$ for these values.

67. Size, chromatic number, and connectivity (with J. Bhasker and T. Samad; *Graphs and Combin.* 10(1994), 209–213.)

Let $g(n, k, c)$ denote the minimum number of edges in a k -edge-connected graph having order n and chromatic number at least c (we assume $n > k$ and $n \geq c$). Let (*) denote the condition that there exists a c -critical graph with order n and size $\binom{c}{2} + k(n + 1 - c)/2 - 1$. We prove the following results.

$$g(n, k, c) = \begin{cases} \binom{c}{2} + n - c & \text{if } k = 1 \\ \binom{c}{2} + \lceil k(n + 1 - c)/2 \rceil & \text{if } c > k \geq 2 \text{ and } n \geq c + k, \text{ except:} \\ \binom{c}{2} + k(n + 1 - c)/2 - 1 & \text{if } n > c = k + 1 \text{ and (*)} \\ \lceil kn/2 \rceil & \text{if } c \leq \lceil (k + 1)/2 \rceil \\ \lceil kn/2 \rceil & \text{if } c \leq k \text{ and } n > k + c \end{cases}$$

If $c + k > n > c > k + 1$, then the lower bound $g(n, k, c) \geq \binom{c}{2} + \lceil k(n + 1 - c)/2 \rceil$ holds but may not be best possible. Similarly, for $c + k > n > k > c$, the lower bound of $\lceil kn/2 \rceil$ holds, but constructions with this value are known only for isolated cases. Many of our extremal examples are also k -connected; when this occurs the same result holds with connectivity in place of edge-connectivity.

66. Relaxed chromatic numbers of graphs (with M.L. Weaver; *Graphs and Combin.* 10(1994), 75–93).

Given a family \mathbf{P} of graphs, the \mathbf{P} chromatic number $\chi_{\mathbf{P}}(G)$ of a graph G is the least number of classes in a partition of $V(G)$ such that each class induces a subgraph in \mathbf{P} . Let H' be the class of graphs not containing H , and let H^* be the class of disjoint unions of subgraphs of H . Let P_m, S_m be the path and star with m vertices.

Results: 1) construction of triangle-free k -critical graphs for $\chi_{K'_{r,s}}$. 2) $\chi_P(G) \leq 1 + \max_{F \subseteq G} \delta(F)/k$ when all graphs in \mathbf{P} have minimum degree at least k . 3) $\chi_{S'_{k+1}}(G) \leq \lceil (\Delta(G) + 1)/k \rceil$, and construction of classes of graphs with $\chi_{S'_{k+1}}(G) > \lceil \Delta(G)/k \rceil$. 4) For $m \geq 1$, computation of $\chi_{S_m^*}$ and $\chi_{P'_{m+1}}$ for every complete multipartite graph. 5) Best-possible bound on $\chi_{P'_m}(G)$ in terms of $n(G)$ and $\chi(G)$ (about $\chi(G)/2$ when $n(G)$ is small, to $\chi(G)$ itself when $n(G)$ is large). 6) For the Cartesian product, $\chi_{\mathbf{P}}(F \square G) \leq \max\{\chi_{\mathbf{P}}(F), \chi(G)\}$. 7) Computation of $\chi_{P'_k}, \chi_{P_k^*}, \chi_{S'_k}, \chi_{S_k^*}$ for all products of two cycles and for many products of more cycles.

65. Covering a poset by interval orders (*J. Comb. Theory (A)* 66(1994), 169–171).

At most $\lceil \sqrt{n} \rceil$ interval orders suffice to cover the elements of an n -element poset. More precisely, any poset with at most k^2 elements can be covered by k interval orders, with k interval orders needed if and only if the poset is the disjoint union of k chains of size k .

64. Gray code results for acyclic orientations (with C.D. Savage and M.B. Squire; *Congr. Numer.* 96(1993), 185–204).

The *acyclic orientation graph* of a graph G is the bipartite graph $\text{AO}(G)$ whose vertices are the acyclic orientations of G , adjacent if they differ by reversing one edge. A Hamiltonian cycle of $\text{AO}(G)$ corresponds to a Gray code listing of the acyclic orientations. We construct Gray code listings for the acyclic orientations of odd cycles and chordal graphs. We prove that such listings do not exist for even cycles or for $K_{m,n}$ if $2 \mid mn$ and $m, n > 1$. For the ladder $P_2 \square P_m$ and wheel $K_1 \vee C_m$, Gray code listings of the acyclic orientations exist if and only if m is odd.

63. The total interval number of a graph, I: Fundamental classes (with T.M. Kratzke; *Discrete Math.* 118(1993), 145–156).

A *multiple-interval representation* of a graph G assigns each vertex a union of disjoint real intervals so that vertices are adjacent if and only if the assigned sets intersect. The *total interval number* $I(G)$ is the minimum of the total number of intervals used in any such representation. We obtain the maximum value of $I(G)$ for n -vertex graphs ($\lceil (n^2 + 1)/4 \rceil$), n -vertex outerplanar graphs ($\lfloor 3n/2 - 1 \rfloor$), and connected graphs with m edges ($\lfloor (5m + 2)/4 \rfloor$).

62. Subtree and substar intersection numbers (with Y.-W. Chang, M.C. Jacobson, and C.L. Monma; *Discrete Appl. Math.* 44(1993), 205–220).

The *star number* $s(G)$ [*tree number* $t(G)$] of a graph G is the minimum k such that G is the intersection graph of sets that are unions of k substars [subtrees] of a host tree. Results: 1) $i(G) \leq (\omega(G) - 1)t(G) + 1$, where $i(G)$ is the interval number of G ; this is best possible for $\omega(G) = 1$ and achievable within a factor of $\lg(\omega(G))$ for $\omega(G) > 2$. 2) $s(G) \leq n(G)/3$ (the best-possible bound $s(G) \leq \lceil (n(G) + 1)/4 \rceil$ appears in a later paper). 3) $s(G) \leq i(G)$ for triangle-free graphs. 4) We prove a forbidden subgraph characterization of the “substar graphs” (star number 1). 5) For each k , we construct interval graphs with star number k and substar graphs with interval number k .

61. Generating linear extensions of special posets by adjacent transpositions (*J. Comb. Theory (B)* 58(1993), 58–64).

If P is a rooted forest in which no element has exactly one child, then the linear extensions of P can be successively generated by transpositions of consecutive elements; i.e., P is *adjacent-traversable*. More generally, if P is adjacent-traversable and P' is obtained from P by adding an antichain of size at least two covering (or covered by) a single element of P , then P' is also adjacent-traversable. The proof is constructive.

60. Clique coverings of the edges of the random graph (with B. Bollobás, P. Erdős, and J. Spencer; *Combinatorica* 13(1993), 1–5).

The edges of the random graph (with edge probability .5) can be covered using at most $O(n^2 \ln \ln n / (\ln n)^2)$ cliques. This establishes an upper bound on the intersection number (also called clique cover number) of the random graph. A lower bound, obtained by counting arguments, is $(1 - \epsilon)n^2 / (2 \lg n)^2$.

59. Vertex degrees in planar graphs (with T. Will; in *Planar Graphs*, (W.T. Trotter, ed) *DIMACS Series Discrete Math. Theor. Comp. Sci.* 9(1993), 139–149).

For a planar graph with n vertices we determine the maximum values for the following: 1) The sum of the m largest vertex degrees. 2) For $k \geq 12$, the number and the degree-sum of the vertices with degree at least k . 3) For $6 \leq k \leq 11$, upper and lower bounds for these two values that match for some congruence classes of n . In most cases, the extremal graph G arises from a triangulation H on the set of vertices to have degree k by adding vertices of degree 3 within the faces of H and vertices of degree 2 adjacent to both endpoints of edges of H .

58. Large P_4 -free graphs with bounded degree (with M.-S. Chung; *J. Graph Theory* 17(1993), 109–116).

Let $ex^*(D; H)$ be the maximum number of edges in a connected graph with maximum degree D and no induced subgraph isomorphic to H . This is finite only when H is a disjoint union of paths, in which case we give crude upper and lower bounds. When $H = P_4$, we prove that $K_{D,D}$ is the unique extremal graph. Furthermore, if G is a connected graph P_4 -free graph with maximum degree D and clique number ω , then $e(G) \leq D^2 - D(\omega - 2)/2$.

57. A characterization of influence graphs of a prescribed graph (with G.-C. Chen, R. Gould, M.S. Jacobson, and R. Schelp; *Vishwa Intl. J. Graph Theory* 1(1992), 77–81).

For a set X of vertices in a graph G , the influence graph $I(G, X)$ has vertex set X , with u adjacent to v if and only if $d_G(u, v) \leq d_G(u, X) + d_G(v, X)$. The graphs that arise as influence graphs are precisely those that have a vertex partition into cliques of size at least two. This answers a question of Harary, Jacobson, Lipman, and McMorris.

56. A Graph-Theoretic Game and its Application to the k -Server Problem (with N. Alon, R.M. Karp, and D. Peleg; in *On-Line Algorithms, DIMACS Series Discrete Math. Theor. Comp. Sci.* 7(1992), 1–9 (extended abstract), and *SIAM J. Computing* 24(1995), 78–100 (full paper)).

We study a matrix game on a weighted connected graph. The *tree player* picks a spanning tree T ; the *edge player* picks an edge e . The edge player wins 0 if $e \in E(T)$ and wins $\text{cycle}(T, e)/w(e)$ if $e \notin E(T)$, where $w(e)$ is the edge weight and $\text{cycle}(T, e)$ is the weight of the cycle formed by adding e to T . We prove that the value $\text{Val}(G, w)$ of the game is bounded by $\exp(O(\sqrt{\log n \log \log n}))$ when G has n vertices. We conjecture a bound of $O(\log n)$.

We apply this to the k -server problem on a *road network* represented by the weighted graph G . We present a randomized strategy with competitiveness ratio $k(1 + \text{Val}(G, w))$ against an oblivious adversary. On an n -vertex road network, this yields a randomized algorithm for the k -server problem that is $k \exp(O(\sqrt{\log n \log \log n}))$ -competitive against oblivious adversaries.