REGS Proposal 2010 — Problems in Combinatorics — D. B. West

I propose to continue the very successful REGS program in Combinatorics run every summer since 2004, which has produced about six papers per summer. Many students participate from a range of stages in our PhD program, and typically also a strong handful of students from computer science also participate. The structure for the group is for students to work in teams on a variety of problems. It is partly the large size of the group that makes it successful and leads to many beneficial aspects.

"Problems in combinatorics" is enormously broad, but it befits the breadth of the problems we study in the area and the size of the group of participants. We consider mostly extremal problems and occasionally some structural problems, typically problems where students have a chance to make progress. An extremal problem considers a parameter f defined on a type of combinatorial structure. Over all the structures in a specified family (often specifying the value of some other parameter), we seek the extreme value(s) of f. The classical original extremal problem in graph theory was to seek the maximum number of edges in a simple n-vertex graph having no set of r pairwise-adjacent vertices.

The group will meet throughout Summer Term II, three times per week for 2.5 hours. During the first week, I and some of the students will describe a variety of open problems, and students will begin to discuss them. In even-numbered years, we usually have a one-week hiatus in the formal meetings for the SIAM Meeting on Discrete Mathematics, where additional open problems are collected. Some of the students will attend that conference, and usually others continue to meet here. Usually the meeting occurs during the second or third week of Summer Term II, but in 2010 the meeting is June 14-17, which is the first week. Thus we must either start the REGS program after the meeting or arrange to start during the week before Summer Term II.

The list of problems I will present is not currently available. Most of these problems will be gathered from conferences I will attend during the spring. This has been the method used in the past and has always provided many problems that are relatively new and where students have good chances to make substantial progress.

Extremal problems are well-suited for a project of this nature, because progress can be made by improving known bounds or by solving the extremal problem on special classes of examples (that is, under additional conditions). In addition to relatively new problems, other good problems are those where the full conjecture has resisted attack and become moderately well-known, leaving no expectation of solving it during the summer, but where there are ways to weaken or modify the statement to obtain interesting and more tractable statements. Many structural problems in combinatorics can also be treated this way.

In addition to the problems presented by faculty, each student will also contribute at least one problem. This aspect has worked well in the past, and students find it quite beneficial. Students find problems in journals, on web pages, or at conferences they have attended. To prepare their presentations, students spend time in the library (or on-line) with databases such as MathSciNet to discover what is known about their problem. Along with understanding the known partial results and making a presentation to the rest of the group, these activities contribute to their research training. They also discover that presenting research material to their peers is different from conducting a recitation or class for calculus students; this valuable experience accelerates their readiness to collaborate on and present research.

Because typically more than 20 students now participate each summer, about 40 problems or problem areas are presented each year. Even in the initial sessions, time is reserved for working on the presented problems in small groups. As the summer continues, the group transitions to working only on proving new results. Students choose which problems appeal to them and work together in teams. This activity is not as individually based as thesis research, but it eases the transition to research for students early in their graduate career.

A mix of both experienced and inexperienced students is helpful in a group this large, as some amount of mentoring occurs. In 2009, for the first time, the group included four entering graduate students; they were active participants. Typically, many students continue to participate in years after their REGS funding ends, because they find the experience beneficial; their participation enhances the mentoring for the more junior students. International students also benefit greatly from working with the domestic students, discussing research in English. Participants from computer science have always helped to broaden the perspective on the problems considered; students from each department become familiar with the sorts of questions asked and technique used by the other group.

Participation by individuals who have completed their degrees is also valuable and brings an important outreach component to the program. As the years continue, we are having more and more summer visits from prior students in the program. They return from places around the world to work with our current students.

Valuable for similar reasons is inviting junior faculty from other schools to participate for a couple of weeks, bringing additional research problems and working with our students. This requires additional funding, but the participants return to their own institutions with an appreciation of the program and ideas about how to run similar programs elsewhere. In addition, their viewpoints on problems, approaches, and types of problems provided for the students can be quite different from mine, thereby providing valuable diversification for the students.

In 2007, André Kézdy brought five students from Louisville for one week (unfunded), and since then his students have insisted on holding a similar summer program each year. In 2009, extra funding was available for visiting faculty, but only at the last minute; many good candidates who wanted to come were unable to, but I did find one. Joshua Cooper came from the University of South Carolina for about 10 days and was very enthusiastic about the program. I hope that funding will be available to invite visitors again in 2010.

Meanwhile, I expect to have unfunded visits also from a number of former participants. I do not know all returnees at this point, but I expect visits from Seog-Jin Kim (with one of his students), Stephen Hartke, Hemanshu Kaul, and others.

An additional developing benefit of the program has been the website I have been maintaining since 2008 with web pages for the problems presented. Researchers around the world have been slowly discovering this site and sending additional comments on partial results or other aspects of the problems. In this way the REGS program has been contributing to the research enterprise globally.