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LETTER OF RECOMMENDATION FOR WOJCIECH SAMOTIJ

Wojciech Samotij is probably our quickest-thinking student in combinatorics since Dhruv Mubayi, who graduated in 1998 and then held postdoc positions at Georgia Tech and Microsoft and won a Sloan Fellowship. Like Dhruv, Wojtek is completing his PhD in an unusually short time, a mere three years.

Wojtek has been a delight to have in class. He was attentive and enthusiastic, commenting after class when a particularly beautiful result has been presented. He corrected errors made on the board. When I asked the class for an idea about what to do next in a proof, or an example achieving some conditions, he was almost always ready with the correct answer. He was so far ahead of the rest of the class that I sometimes needed to ask him not to blurt out the answer, as the rest of the class would just wait for him to answer instead of thinking and responding themselves. His homework solutions were much more efficient than other students and sometimes displayed creative approaches I did not know about. He received the only $A+$ in Fall 2007 in the graduate-level introduction to combinatorics, a course considered among our most demanding because it covers a huge amount of material and has substantial weekly homework. He was also at or very near the top score in three other courses I taught.

Wojtek is talented, conscientious, friendly, and reliable. He communicates very clearly and speaks excellent English. His talent and success has been recognized by awarding him fellowships in our department for the past two years; we don't have many fellowships, and it is unusual for a student to receive this much fellowship support. He has presented his joint research with Prof. Balogh in our seminar; it is sophisticated. It is at the cutting edge of research in random graphs, asymptotic enumeration, and stability problems for large graphs. These are very difficult problems, and he speaks with authority about them, displaying a thorough knowledge of related work.

One main focus of their work is an enumerative analogue of classical extremal graph theory. The *Turán number* $ex(n; H)$ of a graph H is the maximum number of edges in an n -vertex graph that does not have H as a subgraph. Taking subgraphs of an extremal graph yields $2^{ex(n; H)}$ graphs with vertex set $[n]$ that do not contain H , and Erdős conjectured that (asymptotically in the exponent) this generates almost all such graphs. Although Erdős, Frankl, and Rödl proved this in 1986 for nonbipartite H , little was known about the bipartite case. Kleitman and coauthors proved the conjecture when H is a 4-cycle (1982) or a 6-cycle (1996). Balogh and Samotij have now proved it for all complete bipartite graphs $K_{s,t}$, a substantial advance after all these years. They also described the structure of almost all graphs not containing $K_{s,t}$ for suitable s and t , which at present seems to be the strongest sort of result sought in this area.

Another investigation concerns a family of graphs having many properties of random graphs. A set of vertices in a graph is *homogeneous* if its elements are pairwise adjacent or pairwise nonadjacent. In the random graph generated with edge-probability $1/2$, the maximum size of a homogeneous set is almost always quite close to $2 \lg n$, where n is the

number of vertices. By Ramsey's Theorem, there is always a homogeneous set of size at least $d \lg n$, for some constant d . A graph is c -Ramsey if it has no homogeneous set of size more than $c \lg n$. Erdős, Faudree, and Sós conjectured in 1997 that the number of distinct pairs $(|V(H)|, |E(H)|)$ achieved by induced subgraphs of any n -vertex c -Ramsey graph is at least some fixed multiple of $n^{2.5}$. Samotij, working with Alon, Balogh, and Kostochka, has obtained $n^{2.369}$ as a lower bound.

Study of the properties of random-like graphs requires thorough understanding of probabilistic methods and random graphs. Among the first results in the evolution of random graphs was the threshold edge probability for the appearance of fixed trees. More recent studies have sought thresholds for the appearance of all trees with a linear number of vertices and bounded maximum degree. Alon, Krivelevich, and Sudakov obtained a sufficient edge probability. Samotij, working with Balogh, Csaba, and Pei, found a simpler proof that gave a much stronger upper bound on the threshold edge probability.

Finally, Samotij and coauthors have further explored the notion of *resilience* of graph properties introduced last year by Sudakov and Vu. The idea is that above the threshold probability for a monotone property, destroying the property may require deleting a lot of edges. Balogh, Csaba, and Samotij showed that in the model with edge probability C/n for an appropriate constant C (in terms of α , γ , and D), breaking the property of containing all trees of order at most $(1 - \alpha)n$ and maximum degree at most D almost always requires deleting a fraction greater than $1/2 - \gamma$ of the edges at some vertex. Their proof uses Kohayakawa's sparse version of the Szemerédi Regularity Lemma, a highly sophisticated recent tool.

These results are notable in solving conjectures or improving results of world-class researchers, using sophisticated techniques and clever arguments from probabilistic and related techniques, and extending the boundaries of what is known about new and difficult questions. Wojtek is a highly capable young mathematician, and I look forward to seeing him develop into a leader in this area.

Sincerely,



Douglas B. West
Professor of Mathematics