

Method for Curve Sketching

Given a function $f(x)$, if we are asked to sketch the curve we have four basic steps we need to undertake in order to produce a fairly accurate sketch.

- Compute the first and second derivatives, $f'(x)$ and $f''(x)$.
- Use the computed derivatives to determine for what values of x $f(x)$ is increasing ($f'(x) > 0$), decreasing ($f'(x) < 0$), concave up ($f''(x) > 0$) and concave down ($f''(x) < 0$).
- Determine where we will have relative extreme points and inflection points. Notice that the extreme points occur when $f'(x) = 0$ and there is a change in the sign of $f'(x)$, that is relative maxima occur when $f'(x) = 0$ and $f(x)$ goes from increasing to decreasing, relative minima occur when $f'(x) = 0$ and $f(x)$ goes from decreasing to increasing. An inflection point occurs when $f''(x) = 0$ and there is a change in concavity.
- Now we use some algebra to find the x -intercepts and y -intercept. The y -intercept is the point $(0, f(0))$ (Note there is only one!), the x -intercepts are points of the form $(x, f(x))$ where $f(x) = 0$.

From this information, we have some very important points we can plot and the derivatives tell us roughly “how to connect the dots.”