Will's Guide To Life, Volume 3

Math 234, Fall 2006

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1 Chapter 6

1.1 Section 6.1

Antiderivatives - given a function $f(x)$ how to get a function $F(x)$ such that $F'(x) = f(x)$.

$$\int x^r dx = \frac{1}{r+1} x^{r+1} + C \qquad\qquad r \neq -1$$
$$\int x^{-1} dx = \ln(x) + C$$
$$\int e^{kx} dx = \frac{1}{k} e^{kx} + C$$

Remember your $+C$'s.

1.2 Section 6.2

The area under the graph can be approximated by a Riemann Sum. For $f(x)$ on $a \leq x \leq b$ we make n subintervals of length $\Delta x = \frac{b-a}{n}$ and on each subinterval pick a point, x_i on the ith subinterval to get the Riemann Sum
$R = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_n)]$.

1.3 Section 6.3

The definite integral is the limit of the Riemann Sums as $n \to \infty$. The definite integral gives you the algebraic area (area under the x- axis is negative) under the curve $f(x)$ on $a \leq x \leq b$. The Fundamental Theorem of Calculus relating integrals and derivatives. For $f(x)$ continuous on $a \leq x \leq b$, if $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

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1.4 Section 6.4

Area in the xy-plane. Definite integrals play nice with sums, differences and multiplying by a constant. The area between two curves can be found using a definite integral. (Make sure you justify which curve is bigger on your interval!) If $f(x) \leq g(x)$ on $a \leq x \leq b$, the area between the two curves is

$$\int_a^b [g(x) - f(x)]dx$$

1.5 Section 6.5

The average value of $f(x)$ on $a \leq x \leq b$ is $\frac{1}{b-a}\int_a^b f(x)dx$. The consumer's surplus is the area between two specific curves. For a commodity with demand $f(x)$ with demand A and $F(A) + B$ the consumer's surplus is

$$\int_0^A [f(x) - B]dx$$

2 Chapter 9

2.1 Section 9.1

Integration by substitution. Looking at an integral and finding a function $u(x)$ such that the integrand looks some function of u multiplied by du.

2.2 Section 9.2

Integration by parts.

$$\int f(x)g(x)dx = f(x)G(x) - \int f'(x)G(x)dx$$

Picking $f(x)$ so that $f'(x)$ is simpler than $f(x)$ and then finding $G(x)$, the antiderivative of $g(x)$.

2.3 Section 9.6

Improper integrals. One of the limits of integration is infinite, either ∞ or $-\infty$. Have to evaluate using a limit as functions are not defined at infinity. For example,

$$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$
$$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty}\int_a^b f(x)dx$$

3 Chapter 7

3.1 Section 7.1

Functions of several (usually two) variables.

3.2 Section 7.2

Partial derivatives. Functions of more than one variable do not have a derivative per se, they have partial derivatives. $f_x(x,y)$ takes the derivative of $f(x,y)$ with respect to x treating y as a constant. $f_y(x,y)$ takes the derivative of $f(x,y)$ with respect to y treating x as a constant. Geometrically, $f_x(x,y)$ gives a tangent line to the surface $f(x,y)$ "parallel" to the x-axis. Similarly $f_y(x,y)$ gives a tangent line "parallel" to the y-axis.

3.3 Section 7.3

Maxima and Minima of functions of several variables. For $f(x,y)$ to have a min or max at (a,b), we must have both $f_x(a,b) = 0$ and $f_y(a,b) = 0$. This does not guarantee (a,b) is a min or max, for that we need to use the Second Derivative Test. $D(x,y) = (f_{xx}(x,y))(f_{yy}(x,y)) - (f_{xy}(x,y))^2$, if $D(a,b) = 0$, we know nothing. If $D(a,b) < 0$, (a,b) is neither a min nor a max. If $D(a,b) > 0$, we have to look at $f_{xx}(a,b)$, if $f_{xx}(a,b) < 0$, (a,b) is a min, if $f_{xx}(a,b) > 0$, (a,b) is a max.

3.4 Section 7.4

Constrained optimization. Given that we want to optimize $f(x,y)$ subject to $g(x,y) = 0$, first build $F(x,y,\lambda) = f(x,y) + \lambda(g(x,y))$. Then take partial derivatives $F_x(x,y,\lambda)$ and $F_y(x,y,\lambda)$, set them equal to zero and solve both for lambda. Set the two expressions for λ equal to each other and solve to find x in terms of y (or y in terms of x). Take this expression and plug it into the constraint equation $g(x,y) = 0$ and solve to get values of x and y. Note we do **not** use the Second Derivative Test.

3.5 Section 7.7

Double Integrals. $\int\int_R f(x,y)dxdy$ gives us the volume of a solid of height $f(x,y)$ over the region R in the xy-plane. Iterated integrals are double integrals in which the inner integral has functions of one variable as the limits of integration rather than numbers. We deal with these by first considering the inner integral, if the integral is with respect to y (if it has dy first), it sees x as a constant, similarly if the integral is with respect to x, it sees y as a constant. We then evaluate that integral to yield an integral in terms of only one variable.