Limits at Infinity

Now, if we want to evaluate limits as $x \to \infty$, we have the more rigorous way and a more intuitive way to look at these limits. So, the basic example we have is something like:

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)}$$

Where $P(x)$ and $Q(x)$ are polynomials. Intuitively, we see that as x gets "really big," the only term in a polynomial that will matter is the highest power of x. So, if the highest power of x in $P(x)$ is n, and the highest power of x in $Q(x)$ is m, then the limit will go to zero if $m > n$ and go to infinity if $n > m$. Now, if we have $m = n$, then the limit will be the coefficient of x^n in $P(x)$ divided by the coefficient of x^m in $Q(x)$.

The more rigorous way say that we take the highest power of x we see, in the top and bottom, and divide through every term on the top and bottom by that power of x. The net effect is that everything goes to zero as $x \to \infty$ except for the coefficients on the highest power of x.

Examples:

$$\begin{aligned}
\lim_{x\to\infty} \frac{3x^2 - 4x + 7}{5x^3 - 9} &= \lim_{x\to\infty} \frac{\frac{3x^2}{x^3} - \frac{4x}{x^3} + \frac{7}{x^3}}{\frac{5x^3}{x^3} - \frac{9}{x^3}} \\
&= \lim_{x\to\infty} \frac{\frac{3}{x} - \frac{4}{x^2} + \frac{7}{x^3}}{5 - \frac{9}{x^3}} \\
&= \frac{0 - 0 + 0}{5 - 0} = 0
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to\infty} \frac{x^2 + 5x + 8}{x - 8} &= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} + \frac{5x}{x^2} + \frac{8}{x^2}}{\frac{x}{x^2} - \frac{8}{x^2}} \\
&= \lim_{x\to\infty} \frac{1 + \frac{5}{x} + \frac{8}{x^2}}{\frac{1}{x} - \frac{8}{x^2}} \\
&= \frac{1 + 0 + 0}{0 - 0} = \infty
\end{aligned}$$