Alternating Series for Math 230

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What is an Alternating Series?

An alternating series is the sum of the terms of an infinite sequence that "jumps back and forth around 0." It usually looks something like this: $\sum_{n=0}^{\infty}(-1)^{n+1}a_n$. We must be careful to distinguish between positive term series and alternating series. We can have an alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ that converges, with the positive term series $\sum_{n=0}^{\infty}|a_n|$ diverging.

Alternating Series Test

If our alternating series is decreasing, that is $a_n \geq a_{n+1} > 0$ and if $\lim_{n\to\infty} a_n = 0$, then the infinite alternating series converges.

Absolute Convergence

A series is said to converge absolutely if the series whose terms are the absolute values of the terms of your given series converges. That is $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\sum_{n=1}^{\infty}|a_n|$ converges.

Furthermore, a series that is absolutely convergent is itself convergent. That is, $\sum_{n=1}^{\infty}|a_n|$ converges implies that $\sum_{n=1}^{\infty} a_n$ converges.

Conditional Convergence

A series is said to conditionally convergent if $\sum_{n=0}^{\infty} a_n$ converges, but $\sum_{n=0}^{\infty}|a_n|$ diverges.

A good example of this is the series $\sum_{n=0}^{\infty}\frac{(-1)^n}{n}$. This series converges by the Alternating Series Test, but $\sum_{n=0}^{\infty}|a_n| = \sum_{n=0}^{\infty}\frac{1}{n}$, which diverges.

The Ratio Test

Given a series $\sum_{n=1}^{\infty} a_n$, we define ρ by:

$$\rho = \lim_{n \to \infty} |\frac{a_{n+1}}{a_n}|$$

Our series converges absolutely if $\rho < 1$, it diverges if $\rho > 1$ and if $\rho = 1$, it tells us nothing and we have more work to do.

The Root Test

Given a series $\sum_{n=1}^{\infty} a_n$, we define ρ by:

$$\rho = \lim_{n \to \infty} |a_n|^{\frac{1}{n}}$$

Our series converges absolutely if $\rho < 1$, it diverges if $\rho > 1$ and if $\rho = 1$, it tells us nothing and we have more work to do.

Terms That Call for Certain Tests

If our series $\sum_{n=1}^{\infty} a_n$ has certain distinguishing features, we usually can apply a certain test.

Form Appearing in Sum	Test to Use
$n!$	Ratio Test
a^n where a is any number	Root Test