Infinite Sequences for Math 230

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What is a Sequence?

A sequence is little more than a list of numbers. We can also view a sequence as a function f from the natural numbers[1], **N** to the real numbers with $f(n) = a_n$. A sequence is usually denoted by $\{a_n\}$.

We say that a sequence converges if we can take the $\lim_{n\to\infty} a_n$ and get a real number. If the limit as $n \to \infty$ is infinite or does not exist, we say that the sequence diverges.

Limit Laws

Given: $\lim_{n\to\infty} a_n = A$ and $lim_{n\to\infty} b_n = B$

$lim_{n\to\infty} c \cdot a_n = c \cdot A$ for any number c.
$lim_{n\to\infty}(a_n + b_n) = A + B$
$lim_{n\to\infty} a_n \cdot b_n = A \cdot B$
$lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$ if $B \neq 0$.

Another idea we will "squeeze" the life out of is the Squeeze Law:

If $a_n \leq b_n \leq c_n$ for all n, and we know that: $lim_{n\to\infty} a_n = lim_{n\to\infty} c_n = A$, then we know that $lim_{n\to\infty} b_n = A$.

A sequence is called monotonic if it is increasing or decreasing. That is $a_{n+1} \geq a_n$ or $a_{n+1} \leq a_n$ for all n. A sequence is called bounded if there exists some number M such that $a_n \leq M$ for all n.
A bounded, monotonic sequence always converges.

[1] The natural numbers, or so called counting numbers are 1, 2, 3, ...