Tests to Determine Convergence[1] for Math 230

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Sections 10.5 and 10.6 outline a few handy methods for determining if and when series converge and diverge. These tests are to be used only on positive term series, that is series of the form $\sum a_n$ where $a_n > 0$.

The Integral Test

When we have a positive term, decreasing series, that is: $an + 1 < a_n$, and we can define a positive-valued, decreasing function f for which $f(n) = a_n$, we can apply the integral test. The integral test says that:
$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx)$ either both converge or both diverge.

The P-Series Test

A very handy test that the book glosses over is the P-Series Test. This says that a series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$ and diverges when $p \leq 1$. We note that the harmonic series is a P-Series with $p = 1$.

The Comparison Test

In the comparison test we take a series that we know either converges or diverges and compare the terms of that series to the terms of the series we are given. Note that we almost always use a p-series, harmonic series or geometric series in this test. We are given $\sum_{n=1}^{\infty} a_n$ and we wish to determine whether or not it converges. So, $\sum_{n=1}^{\infty} a_n$ must be a positive term series and we will compare it to a positive term series $\sum_{n=1}^{\infty} b_n$ that we know converges or another positive term series $\sum_{n=1}^{\infty} c_n$ that we know diverges.
So, if $a_n \leq b_n$ for any n, we know that our given series converges.
If $a_n \geq c_n$ for any n, we know that our given series diverges.

[1] For use on positive term series only

The Limit Comparison Test

So again we are given a positive term series $\sum_{n=1}^{\infty} a_n$ and we wish to know whether it converges or diverges. We compare it to a know positive term series $\sum_{n=1}^{\infty} b_n$. Now, we look at:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

Now, if $0 < L < \infty$, the test tells us that both series either converge or both series diverge. However, if $L = 0$ or $L = \infty$, this test tells us nothing.