

Quadratic Functions in the Denominator Math 230

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Generally we have to pull out a few more tricks to deal when integrating functions that have a quadratic in the denominator. We note that most of the time, these tricks prove easier than resorting to the method of Partial Fractions. The first trick we will have to review is Completing the Square.

Completing the Square:

We have some quadratic function in our denominator, $x^2 + bx + c$ now it is often more useful to have a denominator of the form $u^2 \pm a^2$, so we need to develop some method for changing $x^2 + bx + c$ into something of that form. We first note that: $x^2 + bx + \frac{b^2}{4} = (x + \frac{b}{2})^2$. So remembering our trick of adding zero, we arrive at:

$$x^2 + bx + c = (x^2 + bx + \frac{b^2}{4}) - \frac{b^2}{4} + c = (x + \frac{b}{2})^2 + (c - \frac{b^2}{4})$$

This form is suitably close enough to $u^2 \pm a^2$ that we can go from here to some sort of trigonometric substitution to solve our integral.

We generally want to use this trick only after we have broken down our original integral down and have a part that is of the form $\int \frac{dx}{x^2+bx+c}$. If we have a linear term on the top, we generally want to check for some sort of u , du structure, which would be very nice to integrate.