Intro to Differential Equations for Math 230

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Differential Equations

Differential equations are loosely any equation of a function $f(x)$ defined as some combination of its derivatives. In this class, we will deal with only First Order Differential Equations, that is equations only involving the first derivative of our function. For anyone planning on majoring in any science related field, you will see differential equations for the rest of your life (or education at least.)

Often, differential equations come with an "initial condition," that is a pair (x, y) that satisfy your differential equation. These initial value problems often look like:

$$\frac{dy}{dx} = F(x, y) \qquad y(a) = b$$

Differential Equations as a Function of One Variable

If we are lucky enough to have the derivative expressed as a function of only x or only y, we are in a very good position. From here, we can integrate directly to solve the differential equation. If the derivative is a function of x only:

$$\frac{dy}{dx} = f(x) \qquad y(x) = \int dy = \int f(x)dx$$

If the derivative is a function of y only:

$$\frac{dy}{dx} = g(y) \qquad x + C = \int dx = \int \frac{dy}{g(y)}$$

This equation can often times be solved directly for y, but sometimes it cannot and we must settle for an implicit solution. We note that we are now doing indefinate integration and must remember our $+C$'s.

Special Differential Equations

The Natural Growth Equation:

$$\frac{dx}{dt} = kx, \qquad x(0) = x_0, \qquad x(t) = x_0 e^{kt}$$

The Population Growth model is a case of the Natural Growth Equation. Radioactive Decay is another case of the Natural Growth Equation. In radioactive decay, we often want to know the "half-life" of a substance, or just how much time it takes until half of your starting material has decayed. $t_{\frac{1}{2}} = \frac{\ln 2}{k}$.

The last Special Differential Equation we will look it is Torricelli's Law, which is used in fluid flow. The idea is that if we have a tank with a hole in the bottom, we want to determine just how fast our tank with drain. We learn that the change in volume and the change in the height of a cylindrical tank are both proportional to $\sqrt{y}$. $\frac{dV}{dt} = -c\sqrt{y}$ and $\frac{dy}{dt} = -k\sqrt{y}$.