Polar Coordinates for Math 230

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Polar Coordinates

In our normal Cartesian coordinates, we can express a point as (x, y) where x is the distance from the line $x - 0$, the x-axis and y is the distance from the line $y = 0$, the y-axis. So our Cartesian coordinates are given by two distances. So, imagine that I want to instead express this point with only one distance. We define r to be the distance of our point from the origin $(0, 0)$ using the normal distance function, $r = \sqrt{x^2 + y^2}$.

Now if we just specify a distance from the origin, our new "coordinate system" now will define a set of points of equal distance from the origin, a circle of radius r. We now need one more piece of information about or point, or coordinate, to determine which point we want. So, the best thing we can do is to specify at what angle it is from the x-axis. Let us call this angle θ.

Now, if we look at the triangle that is defined by the x-axis, our angle θ and the height y, we see that we have a nice right triangle, so $\tan(\frac{y}{x} = \theta$ and thus $\theta = \tan^{-1}(\frac{y}{x})$. Now, the information conveyed by r and θ gives us the exact same point as (x, y) in cartesian. We now define the Polar Coordinate system by describing any points in the plane by (r, θ) as described above, with one small distinction. A point can have a "negative radius," that is it will point in the opposite direction of the angle θ with magnitude $|r|$.

This idea of negative radius gives us the following relationships:

$$(r, \theta) = (r, \theta + 2n\pi) = (-r, \theta + \pi) = (-r, \theta + (2n + 1)\pi)$$

We can make the connections between Polar and Cartesian Coordinates by the following equations:

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}(\frac{y}{x})$$

Equations to Get to Know

First of all, a polar equation is usually given by $r = f(\theta)$, r as some function of θ.

There are several common polar equations we should be familiar with. First of all, a circle of radius a centered at the point $(0, a)$ is described by the equation $r = 2a \sin\theta$. And $r = 2a \cos\theta$ is a circle or radius a centered at $(a, 0)$. An equation of the form $r = a(1 \pm \sin\theta)$ or $r = a(1 \pm \cos\theta)$ describe a cardioid, so called because of its heart shape. I will refer you to the text for more pictures and examples.