Will's Guide To Life, Volume 3

Math 181

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Section 13.2

The Knaster Inheritance Procedure. n parties place bids on items to be allocated. The highest bidder receives the object, but have to pay in $\frac{n-1}{n}$ of their bid, and each other party takes out $\frac{1}{n}$ of their bid. The remaining money is split evenly among all the players. Drawback of this procedure is that for valuable items, players need to have large amounts of cash available. Proportional, but not envy-free.

Section 13.3

Taking Turns. Fair-division procedure for exactly two players. They each take turns choosing items. Three main questions for this procedure: how do we decide who goes first? Should the second chooser be compensated? Is it best to use a sincere strategy when choosing items? The bottom-up strategy using preference lists.

Section 13.4

Divide-and-Choose. One party is the divider and the other is the chooser. Strategies can be used if you know how the other player values the items being chosen. Division is both proportional and envy-free.

Logic: Not in the Book

Some basic logic. A $\Rightarrow$ B is equivalent to Not B $\Rightarrow$ Not A. Not equivalent to B $\Rightarrow$ A or Not A $\Rightarrow$ Not B. A if and only if B means that both A $\Rightarrow$ B and B $\Rightarrow$ A are true.

Section 13.5

Cake-Division Procedures: Proportionality. Definitions of proportionality and envy-free for cake division procedures. For two players proportional if and only if envy-free. Envy-free always implies proportional. For more than two players, proportional is easy to find, envy-free is hard to find. Steinhaus Proportional Procedure for Three Players (Lone Divider). Banach-Knaster Proportional Procedure for Four or More Players (Last Diminisher).

Section 13.6

Cake-Division Procedures: The Problem of Envy. Steinhaus and Banach-Knaster are both not envy-free. The Selfridge-Conway envy-free procedure. Main Idea: First find an envy-free allocation for part of the cake (all but some trimmings.) Second, divide the trimmings up in an envy-free manner. You should be able to explain this process!

Modular Arithmetic and Divisibility: Not in the Book

Modular arithmetic. The modulus is the number at which we reset to zero, also the number we divide by and take remainders. $a \equiv b \mod n$ means that $a - b$ is a multiple of n. $a|b$ means there is a whole number c such that $b = ca$. Equivalent to b is a multiple of a. How to tell when a number is a multiple of 2, ends in 0, 2, 4, 6, or 8. A number is a multiple of 4 if and only if the last two digits of the number are a multiple of 4. Tricks for determining if a number is a multiple of 3, 9 or 11. Using these tricks to determine if a number is a multiple of 6, 18, 33, 99, etc.