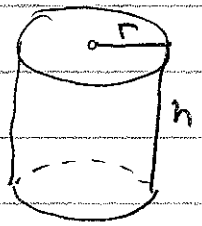
Ex: Minimize the surface area of a cylinder with volume 300π.



$V = \pi r^2 h$

$A = 2\pi r^2 + 2\pi r h$

$300\pi = \pi r^2 h \qquad h = \frac{300}{r^2}$

$A(r) = 2\pi r^2 + 2\pi r\left(\frac{300}{r^2}\right) = 2\pi r^2 + \frac{600\pi}{r}$

domain $(0, \infty)$

$A'(r) = 4\pi r - \frac{600\pi}{r^2} \overset{\text{set}}{=} 0$

$4\pi(r - 150/r^2) = 0$

$4\pi(r^3 - 150) = 0$

$r = \sqrt[3]{150}$

$r = \sqrt[3]{150}$ is in the domain

$A'(r)$ DNE when $r = 0$

$A(0)$ DNE

$A(\sqrt[3]{150}) = 2\pi(\sqrt[3]{150})^2 + \frac{600\pi}{\sqrt[3]{150}}$

$A''(r) = 4\pi + \frac{1200\pi}{r^3}$

$A''(\sqrt{150}) = 4\pi + \frac{1200\pi}{(150)^{3/2}} > 0$

$\therefore$ The surface area is minimized at $300\pi\left(1+\frac{2}{\sqrt{150}}\right)$.