

# Some More of the Fundamental Theorem of Calculus

Math 220

Spring 2009

The Fundamental Theorem of Calculus effectively tells us how to find the anti-derivative of any continuous function.

**Fact:** For any continuous function  $f(x)$  the function

$$\mathcal{F}(x) = \int_a^x f(t) dt$$

is an antiderivative of  $f$ . Here  $a$  is any number for which  $f$  is continuous on  $[a, x]$ .

To use the Fundamental Theorem of Calculus to find the area under the curve, we really need the function  $\mathcal{F}(x)$  defined above to be expressible without the definite integral.

**Example.** If  $f(x) = e^{-x^2}$ , (a continuous function) we can define

$$\mathcal{F}(x) = \int_0^x e^{-t^2} dt$$

to be an antiderivative. However, trying to use the Fundamental Theorem of calculus to calculate the area from  $x = 1$  to  $x = 2$ , would be less than helpful.

$$\begin{aligned} \int_1^2 e^{-t^2} dt &\stackrel{FTC}{=} \mathcal{F}(2) - \mathcal{F}(1) \\ &= \int_0^2 e^{-t^2} dt - \int_0^1 e^{-t^2} dt \\ &= \int_1^2 e^{-t^2} dt \end{aligned}$$

This is not helpful since this says that a definite integral is equal to itself.