

# Will's Guide To Life, Final Edition

Math 220

Spring 2009

## Preliminaries

Remember that the work is more important than the final answer! There is a limit on time, so work hard and work efficiently, do not spend all of your time working any one problem. It is better to have studied too much and be over-prepared than to understudy and do poorly.

You will be expected to know the definitions and statements of the major results and ideas covered in lecture. You need to be able to state **all** hypotheses of the Theorems. In addition, you should know what these Theorems mean. Illustrating a result with a simple picture or graph is a very good thing to be able to do.

The Final Exam is cumulative, you are responsible for the entire semester's worth of material.

## Section 0.1

Lines and slopes. Point-slope and slope-intercept forms of the line. Parallel and perpendicular lines and the relations between their slopes. The precise definition of a function, graph, polynomial, and rational function. The vertical line test. Finding zeroes of functions. Domain and range of a function.

## Section 0.3

Precise definitions of inverse functions and one-to-one functions. The equivalent definitions of one-to-one and how to use them to show a function is one-to-one. The horizontal line test. How to make a function invertible by restricting the domain. Symmetry properties of graphs of inverses.

## Section 0.4

Basic trigonometric functions. Precise definitions of the inverse trigonometric functions. Be very careful with domains! Simplification of compositions of trigonometric and inverse trigonometric functions, always using the reference triangle.

## Section 0.5

Rules of Exponents and Rules of Logarithms. How these rules relate to each other. Precise definitions of exponential and logarithmic functions. Graphs of logarithmic and exponential functions. The change of base formulas and common bases. Solving equations with logarithms and exponentials, always being aware of domain issues. The hyperbolic trigonometric functions.

## Section 0.6

The precise definitions of compositions of functions. The basic transformations of functions and their effect on the graph.

## Section 1.1

Some history, the questions and ideas that led to the development of calculus. The limit as the “fundamental unit” of calculus. Estimating the “slope of a curve” through the use of secant lines. Estimating the length of curves with line segments.

## Section 1.2

The idea of a limit. One-sided limits. What has to happen for a limit to exist. Why the limit does not consider the value of the function at the point. Algebraic manipulations that allow you calculate limits. Using graphs to estimate limits. The rigorous (or precise) definition of a limit (think  $\epsilon$  and  $\delta$ .)

## Section 1.3

The Limit Laws with **all** the hypotheses. When the Limit Laws can fail. Functions that behave nicely with limits. More algebraic manipulations that allow you to calculate limits. The Squeeze Theorem. Limits for piecewise-defined functions.

## Section 1.4

Precise definition for a function to be continuous at a point and on an interval, both closed and open. Removable discontinuities, why this name is appropriate. Functions that are continuous. Continuity rules and their relation to the Limit Laws. How limits and continuous functions play together. The Intermediate Value Theorem, why it intuitively makes sense and its applications.

## Section 1.5

Asymptotes: vertical, horizontal and slant. What it means to say that a limit does not exist,  $= \infty$  or  $= -\infty$ . Remembering that the last two are shorthand and specifically what they mean. Limits at infinity. Indeterminate forms and how to deal with them algebraically.

## Section 2.1

Tangent lines as the limit of secant lines. Instantaneous velocity as the limit of average velocity on smaller and smaller intervals of time. The similarity between the formulas for the slope of the tangent line and instantaneous velocity.

## Section 2.2

The derivative. The definition of the derivative and what it means to be differentiable. Calculating derivatives by calculating limits (all calculus comes back to limits!) Prime, Leibniz and “operator” notation for derivatives. The relationship between differentiability and continuity. Ways in which a function can fail to be differentiable.

## Section 2.3

The Power Rule and the General Power Rule. How we can show the Power Rule is true with the Binomial Theorem. The General Derivative Rules, how they are similar to the Limit Laws and how they are different. Higher order derivatives, how to calculate and notation. What higher order derivatives represent physically.

## Section 2.4

Product Rule and Quotient Rules. (This is where derivative rules look different from the Limit Laws.) When you need to use these rules and when it will be easier to simplify the function first.

## Section 2.5

The Chain Rule, how derivatives interact with composition of functions. The different ways to write the Chain Rule (these alternate ways of writing may help you remember it better.) Using the Chain Rule to calculate the derivative of inverse functions.

## Section 2.6

Derivatives and trigonometric functions. The geometric machinery needed to calculate these derivatives and limits. The derivatives of all the trigonometric functions. (Can be derived from the derivatives of sine and cosine and the derivative rules.) Combining these derivatives with the other rules.

## Section 2.7

Derivatives of Exponential and Logarithmic Functions. Why  $e$  is the preferred base of exponential and logarithmic functions. Finding the derivative of the natural logarithmic function by using inverse functions. Logarithmic differentiation.

## Section 2.8

Implicit Differentiation. How to differentiate equations and relations that need not be functions. (This is just using the Chain Rule cleverly.) Using implicit differentiation to find the derivatives of the inverse trigonometric functions.

## Section 2.9

The Mean Value Theorem. (This is a very important result, we will reuse it at the end of the semester in the most important theorem in the class!) Rolle's Theorem, what it says and how it is used in the Mean Value Theorem. The different results that follow from Rolle's and the Mean Value Theorem.

## Section 3.1

Linear Approximations and Newton's Methods. The linear approximation of a function  $f$  at  $x_0$ . Why this is sometimes called the "Tangent line approximation." Increments and differentials, the picture that explains these. Using linear approximations to approximate harder functions, how to pick a good base point. Newton's Method. Where the formula for Newton's Method comes from.

## Section 3.2

Indeterminate Forms and L'Hôpital's Rule. The Indeterminate forms that work for L'Hôpital's Rule. What needs to be true in order to use L'Hôpital. Manipulating other indeterminate forms into forms on which L'Hôpital can be used. Why indeterminate forms are indeterminate.

## Section 3.3

Maximum and Minimum Values. Definitions of absolute and local extrema. The difference between absolute and local extrema. The Extreme Value Theorem, critical numbers and Fermat's Theorem. The relationship between critical numbers and extrema.

## Section 3.4

Increasing and Decreasing Functions. Definition of strictly increasing and decreasing functions, relation to the derivative. Why this relationship is true (MVT). The First Derivative Test.

## Section 3.5

Concavity and the Second Derivative Test. Concave up and concave down, what this means for the function and its derivatives. Inflection points. The Second Derivative Test.

## Section 3.6

Overview of Curve Sketching. Using the results of sections 3.3-3.5 (and previous chapters) to aid in accurately sketching graphs of functions. The important things you need to check to do this.

## Section 3.7

Optimization. How to turn a word problem into a calculus problem, then do the appropriate calculus to solve the problem, and finally give a physical solution to the word problem. Optimizing functions given constraints. This will require some use of geometry and other math skills. Applications in real life where optimization is necessary.

## Section 3.8

Related Rates. Setting up word problems into calculus. Related rates as an application of implicit differentiation.

## Section 4.1

Antiderivatives. The “ $+C$ ” for any indefinite integral, we can find families of antiderivatives, there is never only one. The Power Rule for antiderivatives. Every derivative you learned is an antidifferentiation rule. General properties of antiderivatives. The natural logarithm as an antiderivative.

## Section 4.2

Sums and Sigma Notation. How to set up a basic sum. The index of summation. Some basic sums you need to know in Theorem 2.1. Basic properties of sums. The Principle of Mathematical Induction.

## Section 4.3

Area. Using left-hand and right-hand sums to approximate area under a curve. More rectangles give better approximations. The definition of area under the curve and its relation to Riemann Sums. Why we can pick any point in a subinterval in a Riemann Sum.

## Section 4.4

The Definite Integral. How sums, area under the curve and the integral relate to each other. The definition of the definite integral, how this relates to Riemann Sums. The idea of signed or algebraic area versus total area, why this idea is useful. How to compute definite integrals from the definition. Basic properties of the definite integral. Average values, the formula and what it means physically. The Integral Mean Value Theorem.

## Section 4.5

The Fundamental Theorem of Calculus. How antiderivatives and the area under the curve relate. How antiderivatives and derivatives relate to each other, effectively tying together the two main topics of this course. The area function. Finding derivatives of functions defined by integrals.

## Section 4.6

Integration by Substitution. How this relates to the Fundamental Theorem of Calculus and the Chain Rule. How to choose an appropriate substitution, making sure to completely substitute. The logarithmic integral. Substitution in definite integrals, the two ways to deal with this. Never leaving the limits of integration alone.

## Section 4.7

Numerical Integration. The Midpoint Rule. The Trapezoidal Rule. Simpson's Rule. The reasons why each rule was developed. How they can improve summation. Why we need these rules.

## Section 4.8

The Natural Logarithm as an integral. Defining the natural logarithm by a definite integral. How all the properties of natural logarithms follow from this definition. How this allows us to be sure of the rules we have for differentiating exponential and logarithmic functions.

## Section 5.1

Area Between Curves. Definition and its relation to Riemann Sums and the definite integral. Setting up the integrals correctly, finding points of intersection, keeping track of which function is bigger. Integrating with respect to  $y$  when it is easier.

## Section 5.2

Volume: Slicing, Disks and Washers. Slicing up a solid into tiny slices to make a Riemann Sum with which to compute the volume. Volumes of solids of revolution, looking at what happens to a rectangle from a Riemann Sum for the area. Washers. Revolving around the  $x$ -axis,  $y$ -axis and other lines.

## Section 5.3

Volume by Cylindrical Shells. Computing the volume, looking at what happens to a rectangle from the Riemann Sum for the area. The costs and benefits of this technique versus disks or washers.

## Section 5.4

Arc Length and Surface Area. How to compute arc length, how this comes from a Riemann Sum. Surface area of a solid of revolution, where this formula comes from.

## Section 5.5

Projectile Motion. How to integrate Newton's Second Law of Motion to get equations for projectile motion. Projectiles in two dimensions, how to set up the equations and how to use them (this is a gentle introduction to the idea of vectors.)

## Other Info

- The exam will test both your knowledge of the concepts and ideas presented as well as your ability to work problems.
- Remember that the right work is far more important than the right final answer.
- Be sure to clearly indicate your final answer to a problem by boxing or circling and labeling it as your final answer.
- The best way to study is to re-read your lecture notes and the book, work through the suggested homework problems and look over your graded work. Learn from the mistakes you have made on quizzes and homework, do not repeat them on the exam.

For more Math 220 related information, be sure to check the course website: [www.math.uiuc.edu/~wgreen4/math220\\_spring09.html](http://www.math.uiuc.edu/~wgreen4/math220_spring09.html)