

Math 220 Practice Exam I

Chapters 0 and 1

Problem 1. a) Precisely define the inverse cosine function.

b) Simplify the following expressions $\sec(\cos^{-1} \frac{x}{2})$ and state clearly for which values of x this simplification holds.

Problem 2. For the function $f(x) = x^2 - 1$, find the inverse function and the restricted domain. Clearly explain why you restricted the domain. Finally, graph both functions on the same axes.

Problem 3. a) Solve the following equation for x .

$$\ln(2x^3 - 3x^2 + x) - \ln(x) = 0.$$

b) Express the following expression in terms of one natural logarithm

$$2 \log_3 4 - \log_3 \frac{1}{2}.$$

Problem 4. Explain the relationship between the one-sided limits and the existence of the limit.

Problem 5. Sketch the graph of $f(x) = \begin{cases} x^2 + x + 1 & \text{if } x < -1 \\ 3x + 1 & \text{if } x \geq -1 \end{cases}$, and identify the following limits.

a) $\lim_{x \rightarrow -1^-} f(x)$.

b) $\lim_{x \rightarrow -1^+} f(x)$.

c) $\lim_{x \rightarrow -1} f(x)$.

Problem 6. a) State the Squeeze Theorem.

b) Calculate $\lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right)$.

Problem 7. Given that $\lim_{x \rightarrow a} f(x) = 4$, $\lim_{x \rightarrow a} g(x) = 2$ and $\lim_{x \rightarrow a} h(x) = -4$. Consider $\lim_{x \rightarrow a} \sqrt{[f(x)]^2 + 2h(x) - 4g(x)}$. Evaluate the limit if possible, or explain why you cannot.

Problem 8. Define what it means for a function to be continuous on the interval $[a, b)$.

Problem 9. Explain why each function is discontinuous at the given point by indicating which of the conditions to be continuous are not met.

$$f(x) = \begin{cases} x^2 & x < 2 \\ 3 & x = 2 \\ 3x - 2 & x > 2 \end{cases} \quad \text{at } x = 2$$

Problem 10. Compute the following limits.

a) $\lim_{x \rightarrow \infty} \frac{-3x^3 + 2x^2 + 7x - 8}{2x^3 + 4}$.

b) $\lim_{x \rightarrow 1} \frac{1}{x^2 - 2x + 1}$.

c) $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$.

Problem 11. Find all asymptotes of the function $f(x) = \frac{x^4}{x^3 + 2}$.