

Math 220 Practice Exam III

Chapter 3

Problem 1. a) Calculate the limit $\lim_{x \rightarrow \infty} \frac{\ln x}{e^x}$.

b) The limit in part a) tells us what about the comparative rates at which $\ln x$ and e^x go to infinity? Specifically, which function goes faster?

c) What can we conclude about $\lim_{x \rightarrow \infty} (2e^x - \ln x)$.

Problem 2. Determine where the function is increasing or decreasing, concave up or concave down and classify all extrema and inflection points.

a) $f(x) = \frac{1}{3}x^3 - 5x^2 + 16x + 3$.

b) $g(x) = \ln(x^2 - 1)$.

Problem 3. Use a linear approximation to estimate the following numbers. Be sure to state the functions and value x_0 you employ.

a) $\sin(3.1)$.

b) $\sqrt[4]{80.55}$.

c) $\ln(3)$.

Problem 4. Determine all significant features (as enumerated in class) and sketch the graph of $f(x) = \sin x - \cos x$ on $[0, 2\pi]$.

Problem 5. Suppose that you are blowing up a spherical balloon by adding air at a rate of $\pi \frac{\text{in}^3}{\text{s}}$. What is the rate at which the radius changing when the radius of the balloon is $\frac{32}{3}\pi \text{ in}^3$.

Problem 6. Find the point on the curve $y = x^2$ closest to the point $(3, 4)$.

Problem 7. Suppose a 3 foot long wire is to be cut into two pieces, each of which will be formed into a square. Find the size of each piece to maximize the total area of the two squares.

Problem 8. Calculate the limit:

$$\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^2 + 7}}{\sqrt[3]{x^2 - 4}}.$$

Problem 9. Find all the inflections points of $\tan^{-1}(x^2)$. Be sure to justify your answer.