Math 220 Practice Exam IV

Chapter 4

Problem 1. *Find all functions that satisfy the given conditions.*
a) $f''(x) = \cos x - 2\sin x + 24x$.
b) $f'(x) = \dfrac{4}{\sqrt{1-x^2}} - 3\csc x \cot x + e^x$.

Problem 2. *Compute the following sums.*
a) $\displaystyle\sum_{i=1}^{53}(4i^2 - 5i - 8)$.
b) $\displaystyle\sum_{i=1}^{n}\frac{1}{n}\left[\left(\frac{i}{n}\right)^2 - 2\left(\frac{i}{n}\right) + 7\right]$.

Problem 3. *The sum in part b) of the previous problem can be identified as a Riemann Sum.*
a) It represents an approximation of what function on what interval?
b) Calculate the limit as $n \to \infty$ *of the sum in part b) of the previous problem.*

Problem 4. *From the definition of area under the curve, find the area under the curve* $y = x^2 + 1$ *on the interval* $[0, 2]$.

Problem 5. *The height of a river is given by the function* $h(t) = 4\cos(2\pi t) - 2\sin(2\pi t) + \dfrac{1}{16}t^3 - \dfrac{1}{4}t^4$, *with* t *measured in hours. What is the average height of the river from* $t = 0$ *to* $t = 12$*?*

Problem 6. *Consider the function defined by* $\displaystyle\int_{x^3-3x+5}^{\sin x+14x}\left(e^{t^2}\right)dt$.
a) This is a function of what variable?
b) Compute the derivative.

Problem 7. *Consider the integral* $\displaystyle\int \cos x \sin x\, dx$.
a) Integrate by substitution with $u = \sin x$.
b) Integrate by substitution with $u = \cos x$.
c) Explain what just happened with your answers to the two previous questions.

Problem 8. *Evaluate the following.*
a) $\displaystyle\int_1^2 \frac{1}{\sqrt{x}(\sqrt{x}+1)}\,dx$.

b) $\int e^{\tan t} \sec^2 t \, dt.$

c) $\int_1^4 \frac{x^3 + 7x}{x^3} \, dx.$

Problem 9. *It is a fact that* e^{-x^2} *has no antiderivative that can be defined in terms of elementary functions. Explain one method by which we can find* $\int_0^1 e^{-x^2} \, dx.$

Problem 10. *Evaluate the sum* $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^6}{n^7}$.

(Hint: You do not have a formula for $\sum i^6$*. You must use some other method.)*