Will's Guide To Life, Volume 4: Final Edition

Math 231

Fall 2008

Section 6.1

How to use simple substitutions to get an integral to look like the form that we find in an integral table. Learn the integration rules in the table on p. 510 of the book. (The last 4 can be done via trig tricks.)

Section 6.2

Basic formula for integration by parts: $\int udv = uv - \int vdu$. How to choose u and dv: Choose your dv such that it is actually integrable, that is so that v actually exists. It is a good sign if du is simpler, or at least not as complicated as u. It is a good sign if v is simpler, or at least not as complicated as dv. Reduction formulas. Integrating by parts for a definite integral.

Section 6.3

How to deal with $\int cos^2 x\,dx$ or $\int sin^2 x\,dx$ via the half-angle formulas. The Pythagorean identities. (Only need to remember $\sin^2 x + \cos^2 x = 1$ can derive other two by dividing by either $\sin^2 x$ or $\cos^2 x$.) The different cases of $\sin^m x \cos^n x$ and $\tan^m x \sec^n x$ and our strategies for dealing with them. Remember the trig derivatives. Trig Substitutions for integrals with forms like $\pm a^2 \pm u^2$. The three different trig substitutions and drawing a reference triangle to get back to the correct variable. (The table on p. 528 of the book is quite nice.)

Section 6.4

How to deal with Rational Functions. Using long division if the degree of the numerator is greater than the degree of the denominator. Tricks to look for before resorting to partial fractions. How to deal with repeated factors or higher order (quadratic, cubic, quartic, etc) in the denominator. Completing the square as a method to turn an "ugly" integral into an easier integral or combination of easier integrals. (This is essentially a tool to turn most any quadratic function into a trig substitution.)

Section 6.6

Improper integrals. A limit of integration is $\pm\infty$ or there is some infinite discontiniuity at some point inside the limits of integration. Dealing with a limit of integration of $\pm\infty$ by taking limits. We deal with an integral that has an infinite discontinuity on the interval by again taking limits. The ideas of diverging to infinity and diverging by oscillation. The p-test for convergence of $\int \frac{1}{x^p}\,dx$ around 0 and ∞. Using the comparison test to show an integral converges or diverges.

Section 7.1

Growth or decay problems. The differential equation they satisfy, the growth constant k, half-life and how they relate. Radioactive decay. Newton's Law of Cooling and compound interest. Finding the general and particular solutions of "nice" differential equations.

Section 8.1

The idea of an infinite sequence. Convergence and divergence of sequences, in particular note that sequences can diverge to infinity or diverge by oscillation. Using the "Squeeze Theorem" to find limits. Representing a sequence as a function from the whole numbers to the real numbers. L'Hôpital's Rule for sequences, and how to use it and why it doesn't apply to sequences directly. The definition of convergence for a sequence (ϵ and N). Bounded and monotonic sequences and how to show a sequence is bounded or monotonic.

Section 8.2

Infinite series, make sure you remember the difference and connection between sequences and series. The limit of partial sums to find the sum of an infinite series. The geometric series, $\displaystyle\sum_{n=0}^{\infty} ar^n$ and the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. The geometric series converge to $\frac{a}{1-r}$ if $|r| < 1$, the harmonic series diverges. The k^{th} Term Test for Divergence, what it tells us, what it <u>doesn't</u> tell us.

Section 8.3

The Integral Test, given a decreasing, positive term series and a function, $f(n) = a_n$, where a_n is the n^{th} term in the series. How we can relate the convergence or divergence of the infinite series $\sum_{n=1}^{\infty} a_n$ to the convergence or divergence of the improper integral $\int_1^{\infty} f(x)dx$. The p-Series, $\sum_{n=1}^{\infty} \frac{1}{n^p}$, converges for $p > 1$, diverges for $p \leq 1$. Comparison Test for Positive Term Series. Compare a series to a known series such as a geometric series or p-Series term-by-term to see if it converges or diverges. Limit Comparison Test, using limits (what a series "looks like" for large n) and a known series to determine whether a series converges or diverges.

Section 8.4

Alternating Series, a positive term series with a $(-1)^n$ thrown in to make it alternate about zero. The Alternating Series Test, a decreasing series that approaches zero as n approaches infinity converges, but we do not know from this test whether it is absolutely or conditionally convergent.

Section 8.5

The difference between absolute convergence and conditional convergence, always remembering to look at the series of absolute values. If the series of absolute values converges, the series converges absolutely. Ratio and Root Tests. What the different values of the limit Ł tell us, particularly that $L = 1$ tells us nothing.

Section 8.6

Power Series. The definition of a power series, and the idea of a Radius of Convergence and Interval of Convergence. Finding the Interval of Convergence by setting up a Ratio Test and setting it less than 1, then checking our endpoints to see if the series will converge there. Integrating and differentiating power series term-by-term when we are in the interval of convergence.

Section 8.7

Taylor Series. The general form for the k^{th} term of a Taylor Series. Taylor Polynomials as partial sums of the Taylor Series. Taylor's Theorem, with all the hypotheses. What is z in the remainder term? Using Taylor's Theorem to prove certain Taylor Series converge. You need to know the Taylor Series for

$\sin x$, $\cos x$, e^x, $\ln|1-x|$ and $\arctan x$. Know how to calculate the interval of convergence for Taylor Series.

Section 8.8

Applications of Taylor Series. Using Taylor Polynomials to approximate functions at some point. Using known Taylor Series to create power series expansions for other functions. Making sure to check the interval of convergence! Using series expansions to calculate limits instead of using L'Hôpital. Using Taylor Polynomials to approximate definite integrals. Representing integrals with Power Series. The Binomial Series.

Section 8.9

Fourier Series. The definition of the Fourier Series. The formulas for the Fourier Coefficients for a 2π periodic function on $[-\pi, \pi]$. Tricks you can play with even and odd functions. Extensions of the Fourier Series to L periodic functions.

Section 9.1

Plane Curves and Parametric Equations. Orientation of a parametric curve, one way it has more information than the graph $y = f(x)$. Using parametric equations to make graphs in the $x-y$ plane that are not functions. Defining circles and ellipses by parametric equations. Eliminating the parameter in a parametric equation to get $y = f(x)$. (Being careful here!) The difference between graphs intersecting and parametric curves intersecting.

Section 9.2

Calculus and Parametric Curves. How to find the slope of the tangent line, $\frac{dy}{dx}$ for a parametric curve. When to use L'Hôpital, when you can have a vertical tangent line. How to calculate the second derivative $\frac{d^2y}{dx^2}$ and how NOT to. The speed of a parametric curve. Area enclosed by a parametric curve, it's different between traversing a curve clockwise or counterclockwise!

Section 9.3

Arc Length and Surface Area in Parametric Equations. The formula for the arc length of a parametric curve, how it relates to the arc length of a curve $y = f(x)$. The formula for the surface area of a curve rotated around a line. How this formula relates to the arc length.

Section 9.4

Polar Coordinates. How using a radius r and an angle θ give you the same information as (x, y) does. Converting from Cartesian (x, y) coordinates to Polar (r, θ). Graphing simple polar functions such as circles, lines through the origin, limaçons, roses and cardiods.

Section 9.5

Calculus and Polar Coordinates. Finding $\frac{dy}{dx}$ for a polar equation $r = f(\theta)$. Horizontal and vertical tangent lines to polar equations. Finding the area enclosed by a curve in polar coordinates. How the area integral comes from "infinitessimal triangles." How to find the area between to polar curves. Intersections of polar curves. Arc length for a polar curve (it has a fairly nice formula.)

Section 9.6

Conic Sections. The six conic sections: circle, ellipse, hyperbola, parabola, line and point. The focus and directrix of a parabola. How conic sections can be defined by a distances from points or points and lines. The foci, center and axes of an ellipse. The center and vertices of an hyperbola. Asymptotes of hyperbolas.