Merit Worksheet XIV: Infinite Series and Convergence Tests

Section 8.2, 8.3

September 29, 2008

Problem 1. *Determine which of the following series converge or diverge. If they converge, give their sum.*

a) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{3^n}$

b) $\displaystyle\sum_{n=158}^{\infty} \frac{2}{3^n}$

c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1}}{5^n}$

d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{-n}}$

e) $\displaystyle\sum_{n=0}^{\infty} 2^{2n} 3^{1-n}$

Problem 2. *More Geometric Series.*
a) Express $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$ *as a geometric series. (Find* a *and* r*.)*
b) Find the sum of the series in part a).
c) Write the number $1.1\overline{234} = 1.1234234234\ldots$ *as a ratio of integers. (Hint:* $1.1\overline{234} = 1.1 + \frac{234}{10^4} + \frac{234}{10^7} + \ldots$*.)*
d) Similar to part c), show that $.\overline{999} = 1$.

Problem 3. *The* k^{th} *Term Test.*
a) State the k^{th} *term test.*
b) Explain why this theorem makes sense.
c) Give an example of a series whose terms go to zero but still diverges.
d) Can you use the k^{th} *Term Test to prove a series converges? Prove or give a counterexample.*

Problem 4. *Determine whether or not the following series converge.*

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 - 4n + 1}{14n^3 + 15n^2 + 5}$

b) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

c) $\sum_{n=1}^{\infty} \frac{3^{3n}}{28^n}$

Problem 5. $\infty - \infty = ?$
Theorem 2.3 in the book doesn't mention anything about adding two divergent series.
a) Give two examples of divergent series $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *such that their sum* $\sum_{n=1}^{\infty}(a_n + b_n)$ *diverges.*
b) Give two examples where the sum converges.
c) Is it possible that the sum of two convergent series diverge?

Problem 6. *Determine the convergence of the following series.*

a) $\sum_{n=1}^{\infty} \frac{1}{3 - 2\cos n}$

b) $\sum_{n=1}^{\infty} e^{-n}$

c) $\sum_{n=1}^{\infty} \frac{2^n + 7^n + 4}{11^n}$

d) $\sum_{n=1}^{\infty} \frac{1}{(n \ln^2 n)^{1/3}}$

Problem 7. *Series and Riemann Sums.*
a) Draw a picture of a positive, decreasing continuous function.
b) Draw the picture of the Riemann sum with intervals of length 1 taking left endpoints.
c) Repeat part b) with right endpoints.
d) If a_n *is a positive series and* f *is a positive, decreasing continuous function for* $x \geq 1$ *with* $a_n = f(n)$ *for* $n \geq 1$*, what is the connection between*

$$\sum_{n=1}^{\infty} a_n \qquad \text{and} \qquad \int_1^{\infty} f(x)\, dx$$

Problem 8. *The Integral Test.*
a) State the Integral Test, be sure to note all the hypotheses! (This is well worth memorizing.)
b) Explain why we cannot apply the integral text directly to say that $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ *converges.*
c) Explain, citing another useful theorem, how we can use the integral test indirectly to show convergence.

Problem 9. *p-Series fun.*
a) State the p-Series test.
b) Use the Integral Test and earlier work in improper integrals to prove the p-Series Test.

Problem 10. *State with justification which series converge and which diverge.*

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/4}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

Problem 11. *Find a function* $f(x)$ *such that* $\displaystyle\int_1^{\infty} f(x)\,dx$ *diverges but the corresponding series* $\displaystyle\sum_{n=1}^{\infty} a_n$ *converges. Explain why this does not violate the Integral Test.*

For Next Time

1. Reread section 8.3.
2. Do problems 23-41 odd in 8.2, 1-23 odd in 8.3. Prepare 32 and 38 in 8.2 and 10 in 8.3 to turn in.