Quiz X

Section 7.4, 7.7

November 30, 2006 (Both Sections)

Problem 1. *Find the maximum value of* $x^2 + xy - 3y^2$ *subject to the constraint* $2 - x - 2y = 0$. **Note you are asked to find the maximum value.**

Solution. We have that $f(x, y) = x^2 + xy - 3y^2$ and $g(x, y) = 2 - x - 2y = 0$. We now build $F(x, y, \lambda)$.

$$\begin{aligned} F(x, y, \lambda) &= f(x, y) + \lambda g(x, y) \\ &= x^2 + xy - 3y^2 + \lambda(2 - x - 2y) \end{aligned}$$

We now take partial derivatives of $F(x, y, \lambda)$.

$$\begin{aligned} F_x(x, y, \lambda) &= 2x + y - \lambda \\ F_y(x, y, \lambda) &= x - 6y - 2\lambda \end{aligned}$$

We now set these partial derivatives equal to zero and solve them for λ to get:

$$\begin{aligned} \lambda &= 2x + y \\ \lambda &= \frac{x}{2} - 3y \end{aligned}$$

Setting these different expressions for λ equal to each other we try to find a relationship between x and y.

$$\begin{aligned} 2x + y &= \frac{x}{2} - 3y \\ -4y &= \frac{3x}{2} \\ y &= \frac{-3x}{8} \end{aligned}$$

We now plug this into our constraint equation.

$$\begin{aligned}2 - x - 2(\frac{-3x}{8}) &= 0\\ 2 &= x - \frac{3x}{4}\\ x &= 8\end{aligned}$$

Now, we can plug this back into our relationship between x and y to get that $x = 8$ and $y = -3$.

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Problem 2. *Evaluate*

$$\int_{-1}^{1}\int_{x}^{2x} x + y \; dy \; dx$$

Solution. We first concentrate on the inner integral. It is an integral with respect to y, so it sees x as a constant. So, we have

$$\begin{aligned}\int_{-1}^{1}\int_{x}^{2x} (x + y) \; dy \; dx &= \int_{-1}^{1} (xy + \frac{1}{2}y^2)\Big|_{y=x}^{y=2x} dx\\ &= \int_{-1}^{1} [(x(2x) + \frac{1}{2}(2x)^2) - (x(x) + \frac{1}{2}(x)^2)]dx\\ &= \int_{-1}^{1} [2x^2 + 2x^2 - x^2 - \frac{1}{2}x^2]dx\\ &= \int_{-1}^{1} \frac{5}{2}x^2 dx\\ &= \frac{5}{2}\frac{1}{3}x^3\Big|_{x=-1}^{x=1}\\ &= \frac{5}{6}((1)^3 - (-1)^3)\\ &= \frac{5}{6}(2)\\ &= \frac{5}{3}\end{aligned}$$

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