Quiz II

Section 1.4 - 1.8

September 7, 2006 3pm Section

Problem 1. *Find the first* **and** *second derivatives of*

$$f(P) = (3P+1)^5$$

Solution. We will clearly have to use the Chain Rule (or the "generalized power rule" in section 1.4).

$$\begin{aligned} f'(P) &= 5(3P+1)^4 \cdot \frac{d}{dP}(3P+1) \\ &= 5(3P+1)^4 \cdot (3) \\ &= 15(3P+1)^4 \end{aligned}$$

We now take the derivative of $f'(P)$ to get the second derivative.

$$\begin{aligned} f''(P) &= 15 \cdot 4(3P+1)^3 \cdot (3) \\ &= 180(3P+1) \end{aligned}$$

□

Problem 2. *Use the limit definition of the derivative to compute $f'(2)$ where*

$$f(x) = \sqrt{5-x}.$$

The limit definition is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Solution. Applying the limit definition to our function and applying it to $x = 2$, we have:

$$\begin{aligned} f'(2) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \to 0} \frac{\sqrt{5-(2+h)} - \sqrt{5-2}}{h} \end{aligned}$$

We now decide to multiply by the conjugate on the top and bottom, so in effect we are only multiplying by 1. (A cleverly chosen value of 1 in fact).

$$\begin{aligned} f'(2) &= \lim_{h \to 0} \frac{\sqrt{3-h} - \sqrt{3}}{h} \cdot \frac{\sqrt{3-h} + \sqrt{3}}{\sqrt{3-h} + \sqrt{3}} \\ &= \lim_{h \to 0} \frac{(\sqrt{3-h})^2 - (\sqrt{3})^2}{h(\sqrt{3-h} + \sqrt{3})} \\ &= \lim_{h \to 0} \frac{3-h-3}{h(\sqrt{3-h} + \sqrt{3}} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \to 0} \frac{-h}{h(\sqrt{3-h} - \sqrt{3}} \\ &= \lim h \to 0 \frac{-1}{\sqrt{3-h} + \sqrt{3}} \\ &= \frac{-1}{2\sqrt{3}} \end{aligned}$$

□