Quiz III

Section 3.1-3.2; 2.1-2.4

September 14, 2006

Problem 1. *Compute* $\frac{dy}{dx}$ *using the chain rule formula*

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

where

$$y = \frac{u-1}{u+1} \quad \text{and} \quad u = 2 + \sqrt{x}$$

Solution. We see that we have a nice expression of y in terms of u and u and x. To find $\frac{dy}{du}$ we have to use the quotient rule. The quotient rule says:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Now, applying this to $y(u)$ we have

$$\begin{aligned}\frac{dy}{du} &= \frac{(u+1)(u-1)' - (u+1)'(u-1)}{(u+1)^2} \\ &= \frac{2}{(u+1)^2}\end{aligned}$$

Using the power rule, we have $\frac{du}{dx} = \frac{1}{2}x^{-1/2}$. Now all we need to do is use the chain rule to get:

$$\begin{aligned}\frac{dy}{dx} &= \frac{2}{(u+1)^2} \cdot \frac{1}{2}x^{-1/2} \\ &= \frac{1}{(2+\sqrt{x}+1)^2} \cdot x^{-1/2} \\ &= \frac{1}{\sqrt{x}(3+\sqrt{x})^2}\end{aligned}$$

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Problem 2. *For the curve* $y = x^3 - 3x + 2$*, find all local extreme points and all inflection points (if any). Make sure to justify your answers.*

Solution. To find extreme points and inflection points, we have to start with computing the derivatives.

$$y'(x) = 3x^2 - 3$$
$$y''(x) = 6x$$

Now, we recall that extreme points occur only when $y'(x) = 0$. So, we first set $y'(x)$ equal to zero. $3x^2 - 3 = 0$ gives us $x^2 = 1$ which gives us two possible values of x, $x = 1, -1$. Now, we could check to see if the derivative changes signs at these points, or we can apply the Second Derivative Test. $y''(-1) = -6$ and $y''(1) = 6$, which gives us that there is a relative maximum at $x = -1$ and a relative minimum at $x = 1$. However, we need to find the points $(-1, y(-1))$ and $(1, y(1))$. Evaluating y at these points gives us that our relative extreme points occur at $(-1, 4)$ and $(1, 0)$.

Now, we recall that the inflection points occur only when the second derivative equals zero. So, we set $y''(x) = 0$, this give us that $x = 0$. Now, we need to check that the second derivative actually changes signs at $x = 0$. Since $y''(x) = 6x$ is just a straight line, we see that it does change signs, thus there is an inflection point when $x = 0$. This inflection point occurs at $0, 2)$.

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