Quiz VI

Section 4.6, 5.1 - 5.3

October 15, 2006

Problem 1. *A sample of radioactive material disintegrates from 5 grams to 2 grams in 100 days. After how many days will just 1 gram remain? (Do not attempt to convert your solutions to decimal form.)*

Solution. We first note that radioactive decay tells us the basic form of the equation for $P(t)$, the amount of radioactive material remaining, $P(t) = P_0 e^{kt}$. We need to determine values for the constants P_0 and k. We are told that 5 grams disintegrates to 2 grams in 100 days, so we have the data $P(0) = 5$ and $P(100) = 2$. So, we have $P_0 = 5$ and we have two ways of expressing $P(100)$.

$$\begin{aligned} 2 = P(100) &= 5e^{100k} \\ \frac{2}{5} &= e^{100k} \\ \ln\left(\frac{2}{5}\right) &= 100k \\ \frac{1}{100}\ln\left(\frac{2}{5}\right) &= k \end{aligned}$$

So, we now have a better equation for $P(t)$, $P(t) = 5e^{\frac{1}{100}\ln(2/5)t}$. What we want is the time when $P(t) = 1$, so we have:

$$\begin{aligned} 1 &= 5e^{\frac{1}{100}\ln(2/5)t} \\ \frac{1}{5} &= e^{\frac{1}{100}\ln(2/5)t} \\ \ln\left(\frac{1}{5}\right) &= \frac{1}{100}\ln\left(\frac{2}{5}\right)t \\ 100\frac{\ln(1/5)}{\ln(2/5)} &= t \end{aligned}$$

□

Problem 2. *Use logarithmic differentiation to differentiate*

$$f(x) = (x+1)^4(4x-1)^2$$

(Hint: Take the natural logarithm of both sides, and then differentiate.)

Solution. We will use the fact that $\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$.

$$\begin{aligned} \ln f(x) &= \ln[(x+1)^4(4x-1)^2] \\ &= \ln[(x+1)^4] + \ln[(4x-1)^2] \\ &= 4\ln(x+1) + 2\ln(4x-1) \end{aligned}$$

This is easy to differentiate to get:

$$\begin{aligned} \frac{d}{dx}\ln f(x) &= \frac{d}{dx}[4\ln(x+1) + 2\ln(4x-1)] \\ &= \frac{4}{x+1} + \frac{8}{4x-1} \end{aligned}$$

We now use the fact above to get that

$$\begin{aligned} f'(x) &= f(x)\left(\frac{d}{dx}\ln f(x)\right) \\ &= (x+1)^4(4x-1)^2\left[\frac{4}{x+1} + \frac{8}{4x-1}\right] \end{aligned}$$

□