Quiz VIII

Section 9.1-9.2, 9.6

November 8, 2006

Problem 1. *Evaluate the integral*

$$\int \frac{x}{\sqrt{x+1}}\, dx$$

using integration by parts. (It is possbile that u-substitution is required to compute a certain antiderivative.)

Solution. The first thing we want to do with integration by parts is to choose which function is a good candidate for $f(x)$. In this case, differentiating $(x+1)^{-1/2}$ does not become simpler, but differentiating x gives dx which is simpler. So, we choose $f(x) = x$ which leads to $g(x) = \frac{dx}{\sqrt{x+1}}$, so $f'(x) = dx$. We need to find $G(x)$, the antiderivative of $g(x)$. To do this, let us choose $u = x + 1$ which gives $du = dx$.

$$\begin{aligned}\int (x+1)^{-1/2} dx &= \int u^{-1/2} du \\ &= 2u^{1/2} \\ &= 2(x+1)^{1/2}\end{aligned}$$

Thus, $G(x) = 2(x+1)^{1/2}$, so we can apply our formula for integration by parts to get

$$\begin{aligned}\int \frac{x}{\sqrt{x+1}} dx &= (x)(2(x+1)^{1/2}) - \int (2(x+1)^{1/2})(dx) \\ &= 2x(x+1)^{1/2} - 2\int (x+1)^{1/2} dx\end{aligned}$$

Again using substitution with $u = x + 1$ then $du = dx$. We get

$$\begin{aligned}\int (x+1)^{1/2} dx &= \int u^{1/2} du \\ &= \frac{2}{3} u^{3/2} \\ &= \frac{2}{3}(x+1)^{3/2}\end{aligned}$$

Combing these results, we have

$$\int \frac{x}{\sqrt{x+1}} dx = 2x(x+1)^{1/2} - \frac{4}{3}(x+1)^{3/2} + C$$

□

Problem 2. *Evaluate the following indefinite integral.*

$$\int_2^\infty \frac{1}{(x-1)^{5/2}} \, dx$$

(It is possible that u-substitution is required to compute the antiderivative.)

Solution. We shall indeed use substitution to find the antiderivative. Choose $u = x - 1$ and get $du = dx$.

$$\begin{aligned}\int \frac{1}{(x-1)^{5/2}} dx &= \int u^{-5/2} du \\ &= \frac{-2}{3} u^{-2/3} \\ &= \frac{-2}{3}(x-1)^{-2/3}\end{aligned}$$

So, the Fundamental Theorem of Calculus tell us we can evaluate the definite integral by use of the antiderivative we just calculated.

$$\begin{aligned}\int_2^\infty \frac{1}{(x-2)^{5/2}} dx &= \lim_{b\to\infty} \int_2^b \frac{1}{(x-2)^{5/2}} dx \\ &= \lim_{b\to\infty} \frac{-2}{3}(x-1)^{-2/3}\Big|_2^b \\ &= \lim_{b\to\infty} \frac{-2}{3}(b-1)^{-2/3} - (\frac{-2}{3}(2-1)^{-2/3}) \\ &= \frac{2}{3}\end{aligned}$$

□