Quiz IX

Section 7.1-7.3

November 16, 2006 3pm Section

Problem 1. *Let* $f(x,y) = x^3y + 2xy^2$. *Find* f_{xx}, f_{yy}, *and* f_{xy}.

Solution. To calculate second partial derivatives, we must first calculate first partial derivatives.

$$\begin{aligned}
f_x(x,y) &= \frac{\partial}{\partial x}(x^3y + 2xy^2) \\
&= y\frac{\partial}{\partial x}(x^3) + 2y^2\frac{\partial}{\partial x}(x) \\
&= y(3x^2) + 2y^2(1) \\
&= 3x^2y + 2y^2 \\
f_y(x,y) &= \frac{\partial}{\partial y}(x^3y + 2xy^2) \\
&= x^3\frac{\partial}{\partial y}(y) + 2x\frac{\partial}{\partial y}(y^2) \\
&= x^3(1) + 2x(2y) \\
&= x^3 + 4xy \\
f_{xx}(x,y) &= \frac{\partial}{\partial x}(f_x(x,y)) \\
&= \frac{\partial}{\partial x}(3x^2y + 2y^2) \\
&= 3y\frac{\partial}{\partial x}(x^2) + 2y^2\frac{\partial}{\partial x}(1) \\
&= 3y(2x) + 0 \\
&= 6xy
\end{aligned}$$

$$\begin{aligned}
f_{yy}(x,y) &= \frac{\partial}{\partial y}(f_y(x,y)) \\
&= \frac{\partial}{\partial y}(x^3 + 4xy) \\
&= x^3\frac{\partial}{\partial y}(1) + 4x\frac{\partial}{\partial y}(y) \\
&= 0 + 4x(1) \\
&= 4x \\
f_{xy}(x,y) &= \frac{\partial}{\partial x}(f_y(x,y)) \\
&= \frac{\partial}{\partial x}(x^3 + 4xy) \\
&= \frac{\partial}{\partial x}(x^3) + 4y\frac{\partial}{\partial x}(x) \\
&= 3x^2 + 4y(1) \\
&= 3x^2 + 4y
\end{aligned}$$

□

Problem 2. *Find the relative extrema of*

$$f(x,y) = 2x^2 + y^3 - x - 12y + 7.$$

Make sure you test your critical points!

Solution. To have a relative extrema of $f(x,y)$ at (a,b), we must have that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. So, let us calculate the first partial derivatives.

$$\begin{aligned}
f_x(x,y) &= \frac{\partial}{\partial x}(2x^2 + y^3 - x - 12y + 7) \\
&= \frac{\partial}{\partial x}(2x^2) - \frac{\partial}{\partial x}(y^3) - \frac{\partial}{\partial x}(x) - \frac{\partial}{\partial x}(12y)\frac{\partial}{\partial x}(7) \\
&= 4x - 1 \\
f_x(x,y) &= \frac{\partial}{\partial y}(2x^2 + y^3 - x - 12y + 7) \\
&= \frac{\partial}{\partial y}(2x^2) - \frac{\partial}{\partial y}(y^3) - \frac{\partial}{\partial y}(x) - \frac{\partial}{\partial y}(12y)\frac{\partial}{\partial y}(7) \\
&= 3y^2 - 12
\end{aligned}$$

We now set these equal to zero and see what values of x and y we get. $4x-1=0$ gives us $x=\frac{1}{4}$ and $3y^2+12=0$ gives us $y=\pm 2$. So, we have possible relative extrema at the points $(\frac{1}{4},2)$ and $(\frac{1}{4},-2)$. Now, we need to look at $D(x,y)$ at these points. To determine $D(x,y)$, we need to determine the second partial derivatives.

$$\begin{aligned}
f_{xx}(x,y) &= \frac{\partial}{\partial x}(f_x(x,y)) \\
&= \frac{\partial}{\partial x}(4x-1) \\
&= 4 \\
f_{yy}(x,y) &= \frac{\partial}{\partial y}(f_y(x,y)) \\
&= \frac{\partial}{\partial y}(3y^2-12) \\
&= 6y \\
f_{xy}(x,y) &= \frac{\partial}{\partial y}(f_x(x,y)) \\
&= \frac{\partial}{\partial y}(4x-1) \\
&= 0 \\
D(x,y) &= (f_{xx}(x,y))(f_{yy}(x,y)) - (f_{xy}(x,y))^2 \\
&= (4)(6y) - (0)^2 \\
&= 24y
\end{aligned}$$

We now evaluate $D(x,y)$ at the possible relative extreme points. $D(\frac{1}{4},-2) = 24(-2) = -48 < 0$, so that $(\frac{1}{4},-2)$ is neither a relative minimum nor a relative maximum. Now, $D(\frac{1}{4},2) = 24(2) = 48 > 0$, so we need to do some more work. $f_{xx}(\frac{1}{4},2) = 4 > 0$, so that $(-1,2)$ is a relative minimum.

$\square$