

REVIEW FOR EXAM 1

Notes on the Test: I will mostly be grading your work, not your final answers. This means that I'm fairly generous about small algebra mistakes, but having the correct answer at the end is worth nothing, *a priori*. Here's an example problem, with several graded solutions:

Problem Is $\int_0^1 x^2 dx \leq M_{50}$, where M_{50} denotes a midpoint approximation to this integral? Explain why.

Answer 1. No. (0/10 points for the correct answer)

Answer 2. Yes because $y = x^2$ is concave down on this interval, and so by a theorem M_n is an overestimate. (6/10 points—right idea, but wrong concavity)

Answer 3. No. $\frac{d^2}{dx^2} x^2 = 2 > 0$, so f is concave up. By a theorem M_n is an underestimate where concave up. (10/10 points)

Note that the “correct” answer by itself is worth nothing, and that an “incorrect” answer is worth more than half the points. (Possibly even worth all the points if the circumstance is right.) If I ask you to explain something, and a theorem applies, you don't need to explain why the theorem is true but you do need to explain *why the theorem applies*. In this case, that means computing the second derivative and finding the concavity of the function on the interval $[0, 1]$.

- (1) Let $f(x) = x^3$, $I = \int_1^4 f(x) dx$.
 - (a) Compute L_3 , R_3 , T_3 and M_3 .
 - (b) If possible, write an inequality bounding I between L_{25} and R_{50} and an inequality bounding I between T_{50} and M_{30} . Explain why these inequalities are true.
- (2) If f is a function for which $10^{-10} < f''(x) < 10^{-6}$ on $[1, 3]$, is M_{50} a good approximation to $\int_1^3 f(x) dx$? Explain why.
- (3) Find the arc length of the following curves on the following intervals:
 - (a) $y = x^{3/2}$ on $[1, 2]$.
 - (b) $y = \sqrt{1 - x^2}$ on $[-1, 1]$.
 - (c) $y = 3x + 2$ on $[1, 3]$.
- (4) Find the volume of the following solids:
 - (a) The base of S in the xy -plane is the region bounded by $y = x^2/2$ and the line $y = 2$, and cross-sections of S perpendicular to the x -axis are semicircles.
 - (b) The region bounded by $y = x$, $x = 3$ and the x -axis, revolved around the y -axis.
 - (c) Show that the volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.
- (5) Find the area of region bounded by the curves $y = 4 - x^2$ and $y = x^2 - 4$.
- (6) Solve the following differential equations exactly:
 - (a) $y' = \frac{\sqrt{y}}{1+t^2}$.
 - (b) $y' = ty^3$.
 - (c) $y' = (\sin t)(1 - y)$.
- (7) Apply Euler's method with 3 subdivisions to find $y(4)$ in the following IVP: $y' = y^2$, $y(1) = 2$.
- (8) Find the following integrals:
 - (a) $\int \sin x \cos x \ln \sin x dx$
 - (b) $\int x^5 e^{-x^3} dx$.
 - (c) $\int_1^e x \ln x dx$.
 - (d) Show that $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$, and use this formula to find $\int x^2 e^x dx$.