REVIEW FOR EXAM 3

(1) Which of the following series converge? Fully justify your answer:
   (a) $\sum_{k=1}^{\infty}(-1)^{k+1}/k$
   (b) $\sum_{k=1}^{\infty} 1/k$
   (c) $\sum_{k=1}^{\infty}(-1)^{k+1}\sin k$
   (d) $\sum_{k=1}^{\infty}(-1)^{k+1}/(k^{1/3})$
   (e) $\sum_{k=1}^{\infty} 1/\sqrt{k}$

(2) Give an example of a series that converges, but does not converge absolutely.

(3) Given that $\sum_{k=1}^{\infty} 1/k^2 = \pi^2/6$, find $\sum_{k=1}^{\infty} 1/(2k+1)^2$. (Hint: Evens and odds.)

(4) For a power series centered at 0, which of the following situations are possible? If possible, give an example of a power series with this property. If not, explain why.
   (a) Does not converge at 0.
   (b) Converges only at 0.
   (c) Converges at 7 but does not converge at -10.
   (d) Has interval of convergence:
      (i) $(-4, 5]$
      (ii) $(-4, 4)$
      (iii) $[-4, 4)$
      (iv) $(-4, 4]$
      (v) $[-4, 4]$

(5) Show that $f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$ satisfies the IVP $f'(x) = 2x \cdot f(x)$, $f(0) = 1$. Use this to evaluate f.

(6) (a) Find the power series of $1/(2+x)$ centered at 0 by considering the power series for $1/(1-x)$, and find its interval of convergence.
   (b) Use your answer from part (a) to find the power series centered at 0 for $\ln(2+x)$. Find its interval of convergence.
   (c) Use your answer from part (a) to find the power series for $1/(2+x)^2$. Find its interval of convergence.

(7) Given that $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, and $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, find the first four terms of the power series for $e^x/(1-x)$. (Remember, you can multiply power series just like polynomials.)

(8) Find the Taylor series of:
  (a) $f(x) = e^{2x}$, centered at 0.
  (b) $f(x) = 2\sin x$, centered at 0.
  (c) $f(x) = \ln x$, centered at 1.

(9) Covert the following from polar to rectangular coordinates:
  (a) $(\sqrt{2}, \pi/4)$.
  (b) $(9, 5\pi/4)$.
  (c) $r = 1/(1+\cos\theta)$.
  (d) $r = 5\sec\theta$.
  (e) $r = 2\csc\theta$.

(10) Find the slope of the tangent line at $(\pi/4, \pi/4)$ for $r = \theta$. Find the area of this curve for θ in $[0, \pi]$.

(11) A particle's position at time t is described by $p(t) = (x(t), y(t))$. $p(0) = (1, 1)$, $p'(0) = (2, 3)$, and $p''(t) = (1, t)$. Find expressions for the velocity and position functions. What is the speed of this particle at time $t = 4$?