Lab 2: Rules for bounding the value of the integral

$$\int_a^b f(x)\,dx$$

1. Suppose $f(x)$ is an increasing function. Show geometrically that the left sum will underestimate and the right sum will overestimate.

2. Suppose $f(x)$ is a decreasing function. Show geometrically that the left sum will overestimate and the right sum will underestimate.

3. Is it always true that the value of the indefinite integral is between the left and right sums? Explain or give a counterexample.

4. Suppose $f(x)$ is concave up. Show geometrically that the midpoint sum will underestimate and the trapezoid sum will overestimate.

5. Suppose $f(x)$ is concave down. Show geometrically that the midpoint sum will overestimate and the trapezoid sum will underestimate.

6. Is it always true that the value of the indefinite integral is between the midpoint and trapezoid sums? Explain or give a counterexample.