LAB 3

(1) Let R be a rectangle with vertices at the points $(a, 0), (a, h), (b, 0), (b, h)$, where $0 \leq a < b$ and $h > 0$. If R is rotated about the y-axis, the resulting solid is a *cylindrical shell.* Find a formula for the volume of this solid of revolution in terms of a, b and h.

(2) Let f be a function that is continuous and nonnegative on the interval $[a, b]$, and R be the region bounded above by the curve $y = f(x)$, below by the x-axis, on the left by the vertical line $x = a$, and on the right by the vertical line $x = b$. Furthermore, suppose that $0 \leq a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$, $m_j = (x_{j-1} + x_j)/2$ (midpoint of interval), and that V is the volume of the solid obtained by rotating R about the y-axis.

   (a) Explain why $V \approx \displaystyle\sum_{j=1}^{n} f(m_j)(x_j^2 - x_{j-1}^2)$.

   (b) Show that $f(m_j)(x_j^2 - x_{j-1}^2) = 2m_j f(m_j)(x_j - x_{j-1})$. (*Hint.* $u^2 - v^2 = (u + v)(u - v)$.)

   (c) use parts (a) and (b) to justify the formula $V = 2\pi \int_a^b x f(x) dx$. This formula is called the *method of cylindrical shells.*

(3) Find the volume of a hemisphere with radius R by a quarter-circle of radius R around the y-axis, using the method of cylindrical shells. That is, rotating the region in the first quadrant bounded by $y = \sqrt{R^2 - x^2}$, the x-axis and the y-axis.

(4) Find the volume of the solid of revolution formed when the region bounded by the curve $y = e^{-x^2}$ and the lines $x = 0$ and $x = 1$ is rotated around the y-axis.

(5) (Calculus III in a nutshell) Let S be the solid bounded by the planes $x = a$, $x = b$, $y = c$, $y = d$, the xy-plane and the surface $z = f(x, y)$, where $f(x, y) \geq 0$, $a < b$ and $c < d$.

   (a) Find an expression for A_x–the area of the slice through S with x fixed.

   (b) Find an expression for the volume of S.

   (c) Let $a = c = 0$, $b = d = 1$, and $f(x, y) = x^2 + y^2$. Find the volume of S.

(6) Let R be the region bounded by $y = x^2$ and $y = 8 - x^2$. Find the volume V of revolving this region around the x-axis. (*Hint.* Use the "disks" method, and write V as the difference of two easily calculated volumes. The book calls this method "washers"–see example 5.)