LAB 3

(1) Solve the following differential equations: $\frac{dy}{dx} = \frac{y(x+1)}{x}$, and $xy' = y \ln y$.
(2) Solve the following IVP: $y(0) = 0$, $y' = 1/(2y)$. Check that your solution is correct. Explain how to use Euler's method to approximate sqare roots.
(3) Suppose that $G(y)$ is an antiderivative of $g(y)$, and that $F(t)$ is an antiderivative of $f(t)$. If G has an inverse function $H = G^{-1}$, solve the differential equation $y' = f(t)/g(y)$ (in terms of F, G and H). Check that your solution is correct. (Hint: use the inverse function theorem.)[1]
(4) Show that the following identity holds for all differentiable functions $u(x)$ and $v(x)$ by showing that the derivatives of both sides are equal:

$$\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx.$$

This is called *integration by parts*, and is usually writtent $\int udv = uv - \int vdu$.
(5) Use the method in (4) to find the following integrals:
  (a) $\int \ln x dx$. (Hint: set $u = \ln x$ and $v = x$.)
  (b) $\int \frac{\ln x}{x^2} dx$. (Hint: set $u = \ln x$ and $v = -1/x$.)
  (c) $\int x \sin(3x) dx$. (Hint: set $u = \ln x$, $dv = x^{-2}dx$.)

[1] I have no idea why this was not included in your textbook in one form or another. This problem justifies the use of separation of variables in solving differential equations.