# LAB 5

(1) (a) Evaluate $\int \frac{5x^2-2x+1}{(2x-1)(x^2+1)}dx$, looking at the example on the back page if you get stuck.
    (b) Evaluate $\int \frac{9x^3+9x^2+3x-1}{(x+1)(x-1)(4x^2+4x+2)}dx$
(2) (a) Find the third Maclauran polynomial of $f(x) = x^3 + x^2 + 1$.
    (b) Find the third Taylor polynomial of $f(x) = x^3 + x^2 + 1$ centered at 1.
    (c) Are these two polynomials equal? If so, why? If not, why not?
(3) Recall that $f$ is an even function if $f(-x) = f(x)$ for all $x$ and that $f$ is an odd function if $f(-x) = -f(x)$ for all $x$.
    (a) Show that the derivative of an even function is an odd function. (Hint: use the limit definition of the derivative.)
    (b) Show that the derivative of an odd function is an even function.
    (c) Show that if $f$ is an even function, then $f'(0) = 0$.
    (d) Suppose that $f$ is an even function. Show that any Maclaurin polynomial for $f$ involves only the even powers of $x$.
    (e) Suppose that $f$ is an odd function. Show that the Maclaurin polynomial for $f$ involves only the odd powers of $x$.
(4) Find the fourth Maclaurin polynomial of the following functions:
    (a) $\frac{1}{1-x}$.
    (b) $\ln(1+x)$.
(5) Approximating numbers:
    (a) Find the third Taylor polynomial of $f(x) = \sqrt{x}$ centered at 1. Use this to approximate $\sqrt{2}$. Find the error of approximation.
    (b) Find the fifth Maclaurin polynomial of $f(x) = \arctan x$. Use this to approximate $\frac{\pi}{4} = \arctan 1$. Find the error of approximation.

**Example of partial fractions:** Find the partial fraction decomposition of:

$$\frac{5x^2-2x+1}{(2x-1)(x^2+1)}.$$

Our goal is to write this fraction in the form:

$$\frac{5x^2-2x+1}{(2x-1)(x^2+1)} = \frac{p(x)}{2x-1} + \frac{q(x)}{x^2+1}.$$

$p(x)$ has degree less than the degree of $2x-1$, i.e. $p(x)$ is just a constant $p(x) = A$. $q(x)$ has degree less than the degree of $x^2+1$, i.e. it is a linear polynommial $q(x) = Bx + C$ for some constants $B, C$. In other words, we have (combining fractions):

$$\begin{aligned}\frac{5x^2-2x+1}{(2x-1)(x^2+1)} &= \frac{A}{2x-1} + \frac{Bx+C}{x^2+1} \\ &= \frac{A(x^2+1)+(Bx+C)(2x-1)}{(2x-1)(x^2+1)} \\ &= \frac{(A+2B)x^2+(-B+2C)x+(A+C)}{(2x-1)(x^2+1)}\end{aligned}$$

Since this equation holds, we must have that $5x^2-2x+1 = (A+2B)x^2+(-B+2C)x+(A+C)$, so we know that each coefficient must equal the corresponding coefficient on the other side. In other words, we have the equations:

$5 = A + 2B$; $-2 = -B + 2C$; $A + C = 1$.

We can use high school algebra to solve these equations, yielding $A = 1, B = 2, C = 0$.

$$\frac{5x^2-2x+1}{(2x-1)(x^2+1)} = \frac{1}{2x-1} + \frac{2x+0}{x^2+1}$$

Now if we want to compute the integral of this function, we get

$$\begin{aligned}\int \frac{5x^2-2x+1}{(2x-1)(x^2+1)}dx &= \int \left(\frac{1}{2x-1} + \frac{2x}{x^2+1}\right) dx \\ &= \int \frac{dx}{2x-1} + \int \frac{2xdx}{x^2+1} \\ &= \frac{1}{2}\ln|2x-1| + \int \frac{du}{u} \qquad (u = x^2+1, du = 2xdx) \\ &= \frac{1}{2}\ln|2x-1| + \ln|u| + C \\ &= \frac{1}{2}\ln|2x-1| + \ln|x^2+1| + C.\end{aligned}$$