WORKSHEET FOR 1/26/2009

Reading assignment for Friday. Read 6.2. If you're interested in why Simpson's rule works so well, read: p. 402-406 (optional, but recommended for any engineering majors in the class).

Homework due Friday.

- 5.7: 13, 27
- 6.1: 2, 4, 5

Notes: Some useful formulas:

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

If we break up an interval $[a, b]$ into n equally sized sub-intervals, then there are $n+1$ end-points, so the intervals (each having length $\Delta x = (b-a)/n$) are $[x_0, x_1]$, $[x_1, x_2]$,... $[x_{n-1}, x_n]$, where $x_i = a + i\Delta x$. The midpoint of the i^{th} interval (starting to count at 0, so the last interval is the $(n-1)^{\text{th}}$ interval) is $x_i + \frac{\Delta x}{2}$.

(1) Suppose we are trying to approximate $\int_a^b f(x)dx$. Write down formulas (using Σ-notation for L_n, R_n and M_n. Recall that $T_n = \frac{L_n + R_n}{2}$, and $S_{2n} = \frac{2}{3}M_n + \frac{1}{3}T_n$ (Simpson's rule.

(2) Let $I = \int_0^1 (x^2 + x + 1)dx$

  (a) Write I as a limit of a Riemann sum. (Hint: Use a formula from problem (1), and take the limit as $n \to \infty$).

  (b) Which of the following is true? Explain why.
  - $L_n \leq I \leq R_n$
  - $R_n \leq I \leq L_n$
  - None of the above.

  (Hint: look at example 4 and theorem 1 on p.376-377).

  (c) Which of the following is true? Explain why.
  - $M_n \leq I \leq T_n$
  - $T_n \leq I \leq M_n$
  - None of the above.

  (Hint: look at example 7 and theorem 2 on p.379-380).

  (d) Evaluate the limit in part (a). Do not use the fundamental theorem of calculus: use the formulas above.