

WORKSHEET FOR 1/30/2009

Reading assignment for Monday. Read section 7.1.

Homework due Monday. 7.1: 1, 4, 5, 19 (Hint: look at p.390-391), 29, 30.

Let $I = \int_a^b f(x)dx$. Theorem 3 tells us that if $|f'(x)| \leq K_1$, and $|f''(x)| \leq K_2$ on $[a, b]$, then the following hold:

- $|I - L_n| \leq K_1(b - a)^2/(2n)$
- $|I - R_n| \leq K_1(b - a)^2/(2n)$
- $|I - T_n| \leq K_2(b - a)^3/(12n^2)$
- $|I - M_n| \leq K_2(b - a)^3/(24n^2)$

These represent the worst possible cases (where the derivative or second derivative are as big as possible). For the first two cases, the worst case occurs with a line of slope K_1 , and for the second two, the worst case occurs with a parabola of the form $y = K_2x^2 + bx + c$.

- (1) Let $f(x) = \sin(x^3)$, and let $I = \int_0^\pi f(x)dx$.
 - (a) Find f' , and graph $y = f'(x)$ on the interval $[a, b]$. Approximately where is f' biggest? Smallest? Find a value for K_1 so that you can apply theorem 3. (You don't have to find the smallest value possible for K_1 .)
 - (b) Find L_4 , and use one of the formulas to give bounds: $c_0 \leq I \leq c_1$.
 - (c) Find f'' , and graph it on the interval $[a, b]$. Where is f'' biggest? Smallest? Find a value for K_2 so that you can apply theorem 3.
 - (d) Find M_4 , and use one of the formulas to give bounds: $d_0 \leq I \leq d_1$.
- (2) Let $f(x) = 2x$, and let $I = \int_0^1 f(x)dx$.
 - (a) Calculate I exactly.
 - (b) Find f' , and find the smallest possible bound on the derivative (that is, find the smallest possible value for K_0 such that $|f'(x)| \leq K_0$). What does theorem 3 tell you about the approximation error for R_2 ?
 - (c) Calculate the approximation R_2 for I , and the associated approximation error.
 - (d) Find f'' , and find the smallest possible bound on the second derivative. What does theorem 3 tell you about the approximation error for M_2 ?
 - (e) Calculate M_2 . Draw a picture showing why you got exactly the right answer.